Special Relativity

Theory and Application (Party 2)

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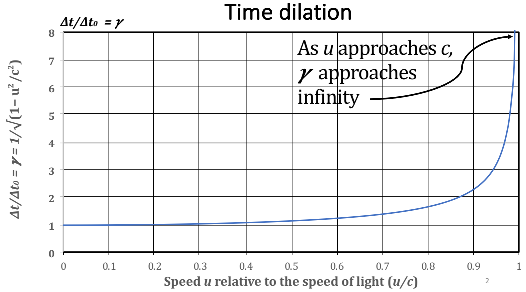
Time dilation



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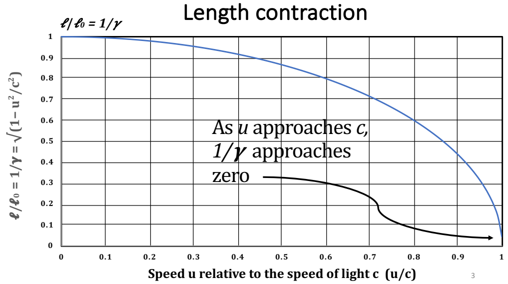
Length contraction



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Relativistic paradoxes

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Given a pair of twins where one travels into space at near the speed of light for say ten years, when the travelling twin returns can they still be the same age?

A train travelling near the speed of light approaches a tunnel which measures 80% of its length when they are stationery relative to each other. Can the train fit into the tunnel?

To answer these and other relativity questions we need to use two important relativistic equations called the ***Lorentz transforms*** named after the Dutch physicist Hendrik Lorentz who developed them and from which Einstein benefitted!

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Lorentz coordinate transformations

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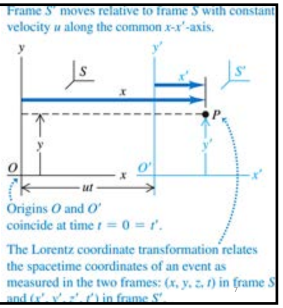
When an event occurs at point (x, y, z) at time t as observed in a frame of reference S, what are the coordinates (x', y', z') and time t' of the event as observed in a second frame S' moving relative to S with a velocity of u in the $+x$ direction?



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Without performing a detailed derivation, the transformation of an event with spacetime coordinates x, y, z, t in frame S and x', y', z', t' in frame S' is done by via the following Lorentz coordinate transformations

$$x' = \gamma(x-ut)$$

$$t' = \gamma(t-ux/c^2)$$

Lorentz coordinate transformations

Where
u is the velocity of S' relative to S in the positive $x - x'$ axis
c is the speed of light and
γ is the Lorentz factor relating frames S and S'

$y' = y$ and $z' = z$ since they are perpendicular to x

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Space and time have become intertwined and we can no longer say that length and time have absolute meanings independent of the frame of reference

Time and the three dimensions of space collectively for a four-dimensional entity called ***spacetime*** and we call x, y, z and t together the ***spacetime coordinates*** of an event

Using the Lorentz coordinate transformations we can derive a set of Lorentz velocity transformations

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$v_x' = (v_x - u)/(1 - uv_x/c^2)$ Lorentz velocity transformation

Where
v_x' is the velocity of object in frame S'
v_x is the velocity of object in frame S
u is the velocity of S' relative to S in the positive direction along the $x - x'$ axis
c is the speed of light in a vacuum

In the extreme case where $v_x = c$ we get

$v_x' = (c-u)/(1-uc/c^2) = c(1-u/c)/(1-u/c) = c$

This means that anything moving at c measured in S is also travelling at c when measured in S' despite the relative motion of the two frames

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When u is less than c, the Lorentz velocity transformation shows that a body with a speed less than c in one frame of reference always has a speed less than c in every other frame of reference

This is one reason for concluding that no material body may travel with a speed greater than or equal to the speed of light in a vacuum, relative to any inertial reference frame

The relativistic generalisations of energy and momentum which we will shortly explore give further support to this hypothesis

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Let's consider an example of the velocity limit which any observer can reach relative to some other observer

If we had a set of five spaceships stacked like Russian dolls where each ship could launch the remaining ships at a velocity equal to the relative velocity of the launching ship as observed from earth what relative velocities could the various ships achieve relative to the earth observer?

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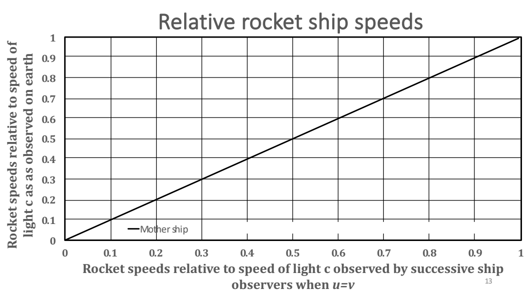

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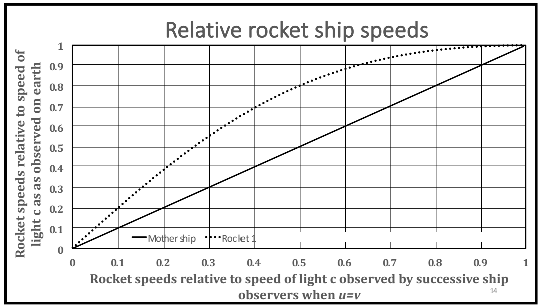

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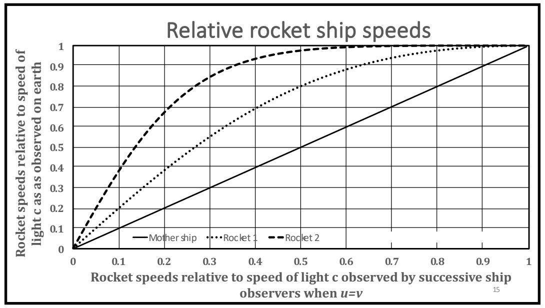

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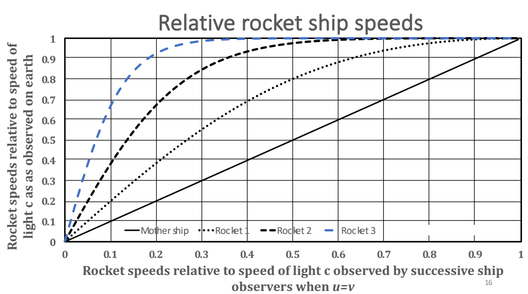

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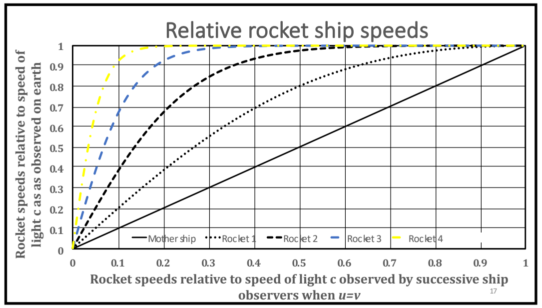

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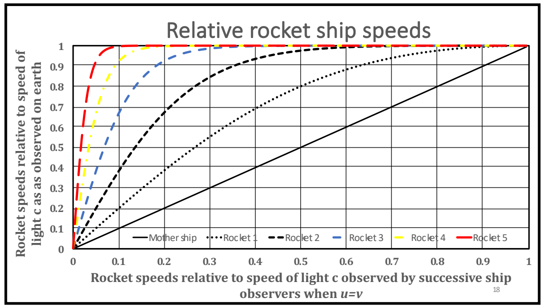

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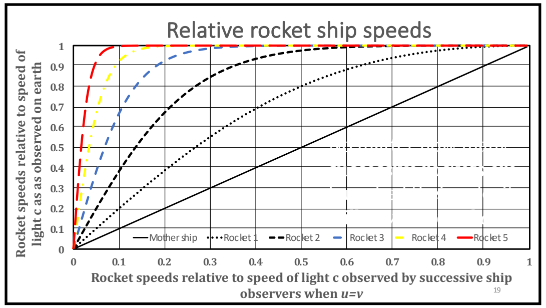

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Relativistic kinematics and the Doppler effect for electromagnetic waves

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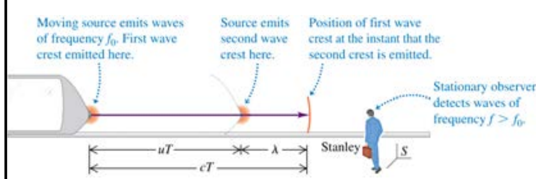

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With an electromagnetic source ***approaching*** an observer the relativistic ***blue shift*** Doppler formula can be derived using the appropriate Lorentz transforms

$$f = \sqrt{(c+u)/(c-u)}\, f_0$$ **Doppler formula (blue shift)**

Where
f = frequency measured by observer
c = speed of light in a vacuum
u = speed of source relative to the observer
f_0 = frequency measured in the rest frame of the source

The doppler blue shift equation indicates that ***f increases*** i.e. the ***wavelength gets shorter*** (bluer) as u approaches the speed of light c

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With light, unlike sound, there is no distinction between motion of source and motion of observer, only the ***relative*** velocity of the two is significant

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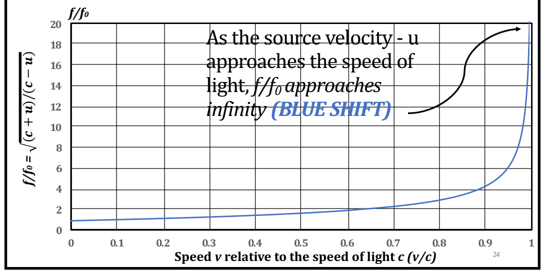
Doppler effect- source approaching observer



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With electromagnetic waves moving ***away*** from an observer the relativistic ***red shift*** Doppler formula can be derived using the appropriate Lorentz transforms

$$f = \sqrt{(c-u)/(c+u)}\, f_0$$ **Doppler formula (red shift)**

Where
f = frequency measured by observer
c = speed of light in a vacuum
u = speed of source relative to the observer
f_0 = frequency measured in the rest frame of the source

The doppler red shift equation indicates that ***f decrease*** i.e. the ***wavelength gets longer*** (redder) as u approaches the speed of light c

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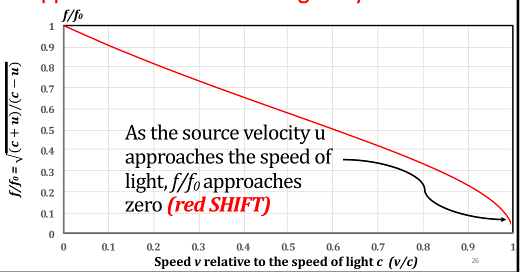
Doppler effect- source moving away from observer



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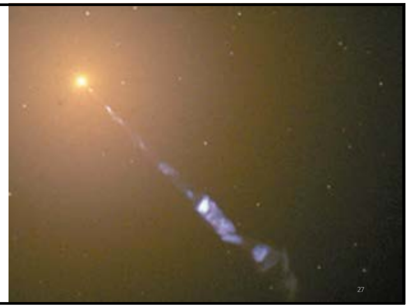
Hubble photograph of a fast moving, Doppler blue shifted jet emanating from a black hole at the centre of Galaxy M87



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Queen Mary 2's radar antennae

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Radar equipment installation at an airport

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Relativistic particle physics

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Relativistic particle momentum $\vec{p}$

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Newton's laws of of motion have the same form in all inertial frames of reference

Using Lorentz transformations to change from one inertial frame to another, the laws should be ***invariant***

The principle of the ***conservation of momentum*** states that when two bodies interact, the total momentum is constant providing that there is no net external force acting on the bodies in an inertial reference frame

Conservation of momentum must therefore be valid in ***all*** inertial frames of reference

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This poses us with a problem: Suppose we look at a collision in an inertial coordinate system S and we find that momentum is conserved

When we use the Lorentz transformation to obtain velocities in a second inertial system S' we find that using the Newtonian definition of momentum ($p = mv$), momentum is not conserved in the second system

To solve this problem we need a more generalised definition of momentum

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The equation will not be derived from first principles, but it will simply be stated below

Suppose we have a material particle with a rest mass of m, when such a particle has a velocity v, then its relativistic momentum $\vec{p}$ is

$$\vec{p} = m\vec{v}/\sqrt{1-(\vec{v}/c)^2} = \gamma m\vec{v}$$ ***Relativistic momentum***

Where
p is momentum
m is the particle (rest) mass
v is the particle velocity
c is the speed of light
γ is the Lorentz factor for a particle
→ denotes a vector quantity

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Relativistic momentum plays a key role in understanding the kinematics of particle physics

Particle velocities will be denoted with *v* for the rest of this presentation

We will no longer be making use of *u*, the relative velocity of reference frames as we will be the stationary observer on earth

Relativistic and Newtonian momentum as a function of relative speed *v/c* are illustrated graphically in the next couple of slide slides

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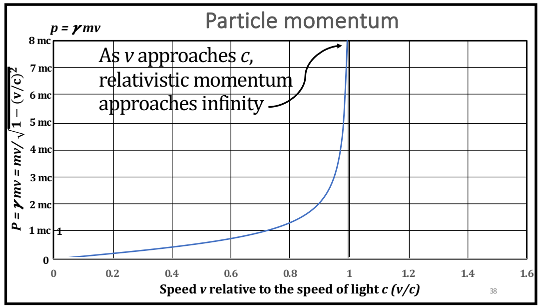

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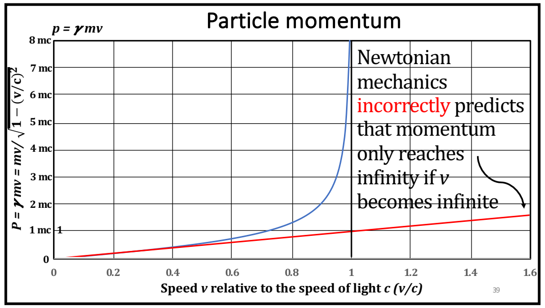

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Force $\vec{F}$ and acceleration $\vec{a}$

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The general form of Newton's second law is

$\vec{F} = d\vec{p}/dt = m\vec{a}$

Experiments show this result is still valid in relativistic mechanics provided we use relativistic momentum. Thus the relativistically correct version of Newton's second law is

$\vec{F} = m\vec{a}/\{\sqrt{1-(\vec{v}/c)^2}\}^3 = \gamma^3 m\vec{a}$ **Force formula**

Where

F is force

m is the particle mass

a is the particle acceleration

v is the particle velocity

c is the speed of light in a vacuum

γ is Lorentz gamma

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Rearranging the previous equation we can establish what happens to the acceleration $\vec{a}$ of a particle of rest mass *m* which is subjected to a constant force

$a = F/m\{\sqrt{1-(v/c)^2}\}^3 = F/m\gamma^3$ ***Acceleration formula***

Where

a is acceleration

F is force

m is the particle rest mass

v is the particle velocity

c is the speed of light in a vacuum

γ is Lorentz gamma

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In Newtonian mechanics if a constant force *F* is applied to a particle of rest mass *m* it will continue to accelerate at a constant acceleration *a* regardless of its speed *v*

In relativistic mechanics, when a particle of rest mass m is subjected to a constant force *F*, its acceleration decreases to zero as its velocity tends toward the speed of light

In fact it does not matter how big the force or nonzero mass is, acceleration will always decrease to zero as the particle speed increases towards the speed of light

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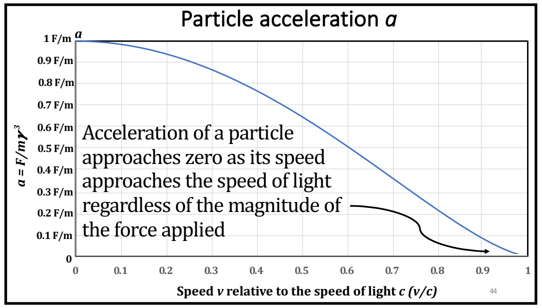

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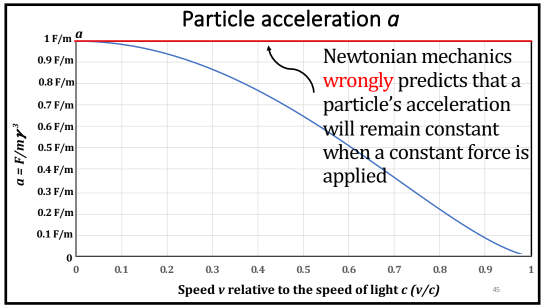

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Relativistic Work and Particle Energy

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The kinetic energy of a particle equals the net energy done on it in moving it from rest to speed v

In ***relativistic*** *terms the* ***kinetic energy K of*** a particle of rest mass m becomes

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$ **Relativistic kinetic energy**

Where

K is the particle kinetic energy

m is the particle rest mass

c is the speed of light in a vacuum

v is the speed of particle

γ is the Lorentz gamma factor relating rest frame of particle and the frame of the observer

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As the speed of the particle v approaches the speed of light so its kinetic energy K approaches infinity

In Newtonian terms K only becomes infinite if v is infinite

The relativistic effects of velocity on the kinetic energy K of a particle ore shown in the next few slides

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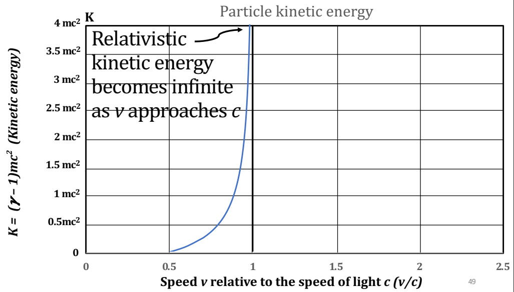

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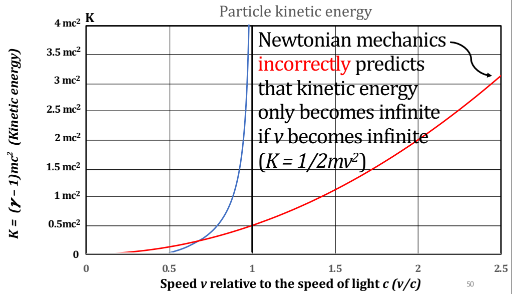

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Total particle energy E, Rest energy (E = mc²) and Massless energy (E = pc)

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To recall, the relativistic kinetic energy equation for a moving particle includes two terms

Motion term Motion independent term

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

The first term depends on motion and the second term is independent of motion

It seems that the kinetic energy of a particle is the difference between some ***total energy E*** and an energy ***mc²*** that it has even at rest

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A particle's total energy ***E*** can thus be expressed as follows

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}} = \gamma mc^2$$ ***Total particle energy***

Where

E is the particle total energy

K is the particle kinetic energy

mc^2 is the particle rest energy

m is the particle rest mass

c is the speed of light in a vacuum

v is the speed of a particle

γ is the Lorentz gamma factor relating rest frame of particle and the frame of the observer

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To summarise, the total energy E of a particle is the sum of its Kinetic energy plus its rest energy

What is apparent is that even when a particle is at rest it still has energy

This is called its rest energy which is proportional to its rest (and only rest) mass

This has been experimentally confirmed. When unstable fundamental particles decay, there is always an energy change consistent with the assumption of a rest energy of mc^2 with a rest mass of m

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The simplest example of the presence of rest energy is the release of energy in the decay of a ***neutral pion*** (***π***).

It is an unstable particle of mass *m* which when it decays (with zero kinetic energy before its decay) releases radiation with an energy exactly equal to $m_\pi c^2$

To put things into perspective, a 50g golf ball has enough rest energy to potentially power a 100 W light bulb for 1.3 million years!

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With a bit of manipulation the momentum and rest energy equations can be reformulated as follows

$$(p/mc)^2 = \frac{v^2/c^2}{1 - v^2/c^2} \quad \text{and} \quad (E/mc^2)^2 = \frac{1}{1 - v^2/c^2}$$

Subtracting and rearranging these equations gives us

$$E^2 = (mc^2)^2 + (pc)^2$$

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For massless particles (m=0)the total energy equation becomes

$$E = pc$$

All ***massless particles*** thus ***travel at the speed of light*** and ***have both energy and momentum***

Photons, the quantum of electromagnetic radiation are massless

The only other known massless particle is the ***gluon***

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The expression also says that for particles at rest (p=0), the total energy equation reduces to

$E = mc^2$ Einstein's famous rest energy equation

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Conservation of mass energy

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From the preceding points it is clear that energy and mass are interchangeable

It is also clear that the principles of conservation of mass and energy should be restated in terms of a broader principle which is ***The law of the conservation of mass and energy***

This law is the fundamental principle involved in the generation of nuclear power. When a uranium or plutonium nucleus undergoes fission in a nuclear reactor, the sum of the rest masses of the resulting fragments is less than the mass of the parent nucleus. An amount of energy is released which equals $E = mc^2$ where m equals the lost mass

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Block 111 Virginia – class nuclear attack submarine

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Fat man replica

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More Relativistic phenomena in nature

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The structure of spacetime is responsible for the force of gravity and the strange idea that the earth is falling in a straight line around the sun!

The sun and all the stars get their energy principally from hydrogen fusion because $E = mc^2$

Cosmic explosions are also driven by $E = mc^2$

In astrophysics the red or blue Doppler shift of celestial bodies tell us how fast stars are approaching or receding from us which has led to our understanding of the expanding universe

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The heat generated by the decay of radioactive elements in the inner layers of the earth provides more than 50% of the heat to keep these layers molten

The movement of tectonic plates depends on having a molten mass on which they can 'float'

This is how our continents and mountains are formed

The earth's rotating molten core also creates the earth's magnetic field which is vital in protecting us from harmful radiation

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Aurora borealis

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Until recently mariners have relied heavily on the magnetic compass for navigation

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You've probably not given it much thought, but the reason why gold is yellow (or rather, golden) is deeply ingrained in its atomic structure and it's because of something called relativistic quantum chemistry

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Simply put, gold's electrons move so fast ($\pm c/2$) in order to avoid being sucked into the nucleus that they exhibit relativistic contraction, shifting the wavelength of light absorbed to blue and reflecting the opposite colour: golden

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These same quantum relativistic effects are also the reason why gold does not corrode easily

The outer electron gets 'trapped' in the inner orbitals nearer the nucleus and is therefore not freely available to react with other elements

In contrast Lithium, which is in the same column in the periodic table, is very reactive

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Similar to gold, mercury is also a heavy atom, with electrons held close to the nucleus because of their speed and consequent mass increase

With mercury the bonds between its atoms are weak, so mercury melts at lower temperatures and is typically a liquid when we see it

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More Practical applications of special relativity

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In particle accelerators many particles have very short half lives. At speeds close to the speed of light half lives are significantly increased giving researchers the opportunity to study them

Modern computer chips. This a little more esoteric, but designing solid-state electronics depends on being able to model electron band structures. That often requires relativistic corrections to do so accurately

In medicine, many body scanners rely on relativistic science for their operation

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Positron emission tomog-raphy (PET) scanner

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Special relativity conclusions

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It may appear that the foundations of Newtonian mechanics have been destroyed. Newtonian mechanics are not wrong, they are simply incomplete. Newton's laws are approximately correct when speeds are small in comparison to *c*

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It may appear that the foundations of Newtonian mechanics have been destroyed. Newtonian mechanics are not wrong, they are simply incomplete. Newton's laws are approximately correct when speeds are small in comparison to *c*

Rather than destroying them, relativity **generalises** them

Even special relativity is not complete!

The **general theory of relativity** goes further and deals with how the geometric properties of space are affected by the presence of matter

Don't forget that all speeds are relative! (Except the speed of light)

You cannot travel faster than the speed of light!

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Recommended you tube viewing by Dr Don Lincoln of Fermilab

- Twin paradox: The real explanation
- What you never learned about mass
- Is relativistic mass real?
- Why you can't go faster than the speed of light
- Relativity: How people get time dilation wrong
- How to travel faster than light
- Einstein's clocks
- Special relativity
- Relativity's key concept – Lorentz gamma
- Why $E = mc^2$ is wrong
- Length contraction: The real explanation

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References

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Special Relativity and Classical Field Theory. The Theoretical minimum by Leonard Suskind and Art Friedman

Richard Feynman – Six not so easy pieces by Matthew Sands

Forces of Nature by Brian Cox and Andrew Cohen

Why does $E = mc^2$? By Brian Cox and Jeff Forshaw

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