THREE REASONS WHY YOU MIGHT NOT UNDERSTAND SPECIAL RELATIVITY

A talk presented to the Macclesfield U3A on 28th April 2015 by Clive Bosman

N.B. Slides with this background colour are not essential to follow the main argument of the talk and were set as ‘hidden’ during the above presentation.

ONLY IF YOU WANT TO KNOW

THE CHARACTERS - TIMELINE

Galilei, Galileo	(b1564)	Italian physicist
Newton, Isaac	(b1642)	English mathematician
Romer, Olaf Christensen	(1676)	Danish astronomer
Bradley, James	(1727)	English astronomer
Fresnel, Augustin	(1814-22)	French engineer
Biot, Jean	(1820)	French physicist
Savart, Felix	(1820)	French physicist
Faraday, Michael	(1831)	English physicist
Doppler, Christian	(1842)	Austrian physicist
Fizeau, Armand	(1849)	French physicist
Foucault, Jean with	(1850)	French physicist
Fizeau, Armand	(1851)	French physicist
Minskowski, Herman	(b1864)	German physicist
Preston, Samuel	(1875)	English engineer
Michelson, Albert	(1881)	American physicist
Morley, Edward	(1887)	American physicist
Fitzgerald, George	(1889)	Irish physicist
Hertz, Heinreich	(1889)	German physicist
Poincare, Henri	(1904)	French physicist
Lorentz, Hendrik	(1904)	Dutch physicist
Pretto, Olinto	(1904)	Italian physicist
Hasenohrl, Fritz	(1904)	Austrian physicist
Einstein, Albert	(1905)	German physicist

We have three basic independent physical properties. (Independent means we cannot add the value of one to another because the result has no physical meaning)

TIME, MASS and LENGTH

What do we understand by these words ?

TIME

Our everyday concept of time duration is a count of the number of repetitions of some regular repetitive process (e.g. swings of a pendulum, oscillations of a balance wheel and spring etc) that take place between two observed events. The count is bundled up into units (e.g. a second).

This is not what is usually meant by TIME in relativity.

MASS

Our everyday concept of mass is a measure of the quantity of matter.

A ton of feathers has the same number of nucleons (protons and neutrons) and electrons as a ton of lead.

This is not what is usually meant by MASS in relativity.

LENGTH

Our everyday concept of the length of a rod is the difference we see between the values of the graduations on a tape measure at the two ends of the rod when placed against it, as on a photograph.

This is not what is usually meant by LENGTH in relativity.

IN A NUTSHELL, WHAT IS SPECEIAL RELATIVITY ABOUT

It’s simply about time and how different observers disagree about the observed duration of time when they observe the same two successive events using identical stopwatches.

This arises in the first place because when we observe an event with one of our senses (e.g. we hear it as in the case of an explosion or we see it as when a light image of the event is incident on our retina) this audio or visual information has taken some time to travel from the event to reach our ears or eyes.

In the case of audio information we are readily aware that there is a travel time as when we shout and wait for the reflection of the sound from the opposite hillside. The sound travels at about 300 m/s.

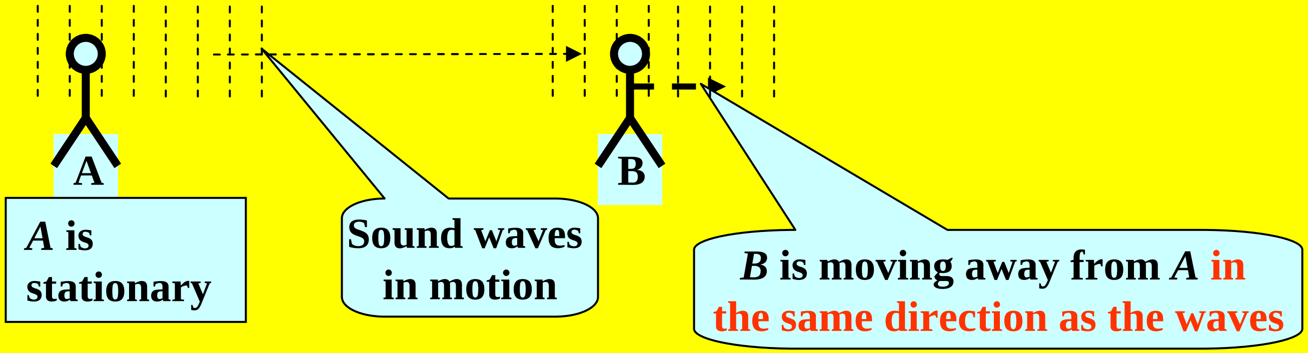
If we flash a torch at a mirror on the opposite hillside, we see the returning image flash almost immediately. The image has a travel time but it is very short. The light image travels at about 300,000,000 m/s. Here the difference between the audio and the visual cases is one only of scale.

We are aware therefore that events occur earlier than when we actually observe them. If we know the speed at which the information travels (say speed c) and how far away the event is (say distance r) we can easily deduce that the event occurred at a time (r/c) earlier than we actually observe it

THIS UNDERSTANDING GIVES RISE TO WHAT WE CALL THE DOPPLER EFFECT AND EVENT TIME

BUT THIS IS ONLY PART OF THE STORY

There is one major difference between the audio and the visual cases.



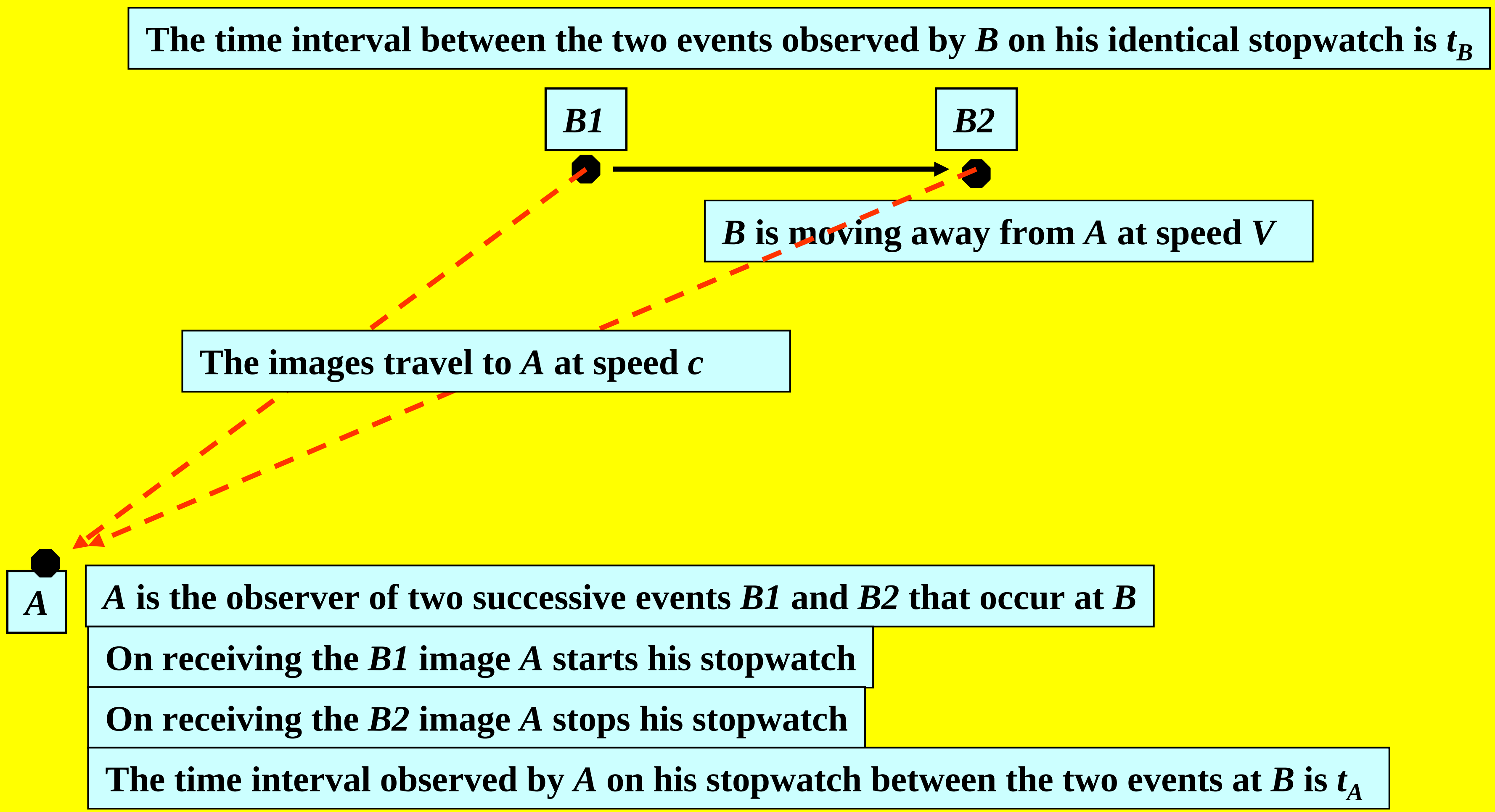
Because *B* is moving in the same direction as the sound waves they will take longer to pass him than they take to pass *A* just as a car overtaking you takes more time to pass you than it does to pass a stationary car.

But if they are light waves, we find that this IS NOT SO. BOTH *A* AND *B* OBSERVE THEM TO TAKE THE SAME TIME TO PASS.

and even if *B* is moving away from *A* at almost the same speed as the light waves, he still finds they take the same time to pass.

When we take into account the fact that light (waves) images travel in vaccuo at the same speed for whomsoever, wherever and under whatever circumstances they are observed, this introduces a further effect called TIME DILATATION, as we shall see.

THE FOLLOWING SCENARIO IS THE KERNEL OF SPECIAL RELATIVITY



The combined effects of DOPPLER and TIME DILATATION result in the following fundamental time relationship between the times t_A and t_B that *A* and *B* actually observe on their stopwatches and from which follow all other deductions regarding length, mass, force, momentum and energy including $E = mc^2$.

$$\frac{t_A}{t_B} = \frac{1 + (V_r/c)}{\sqrt{1 - (V/c)^2}}$$

DOPPLER EFFECT (numerator)

TIME DILATATION (denominator)

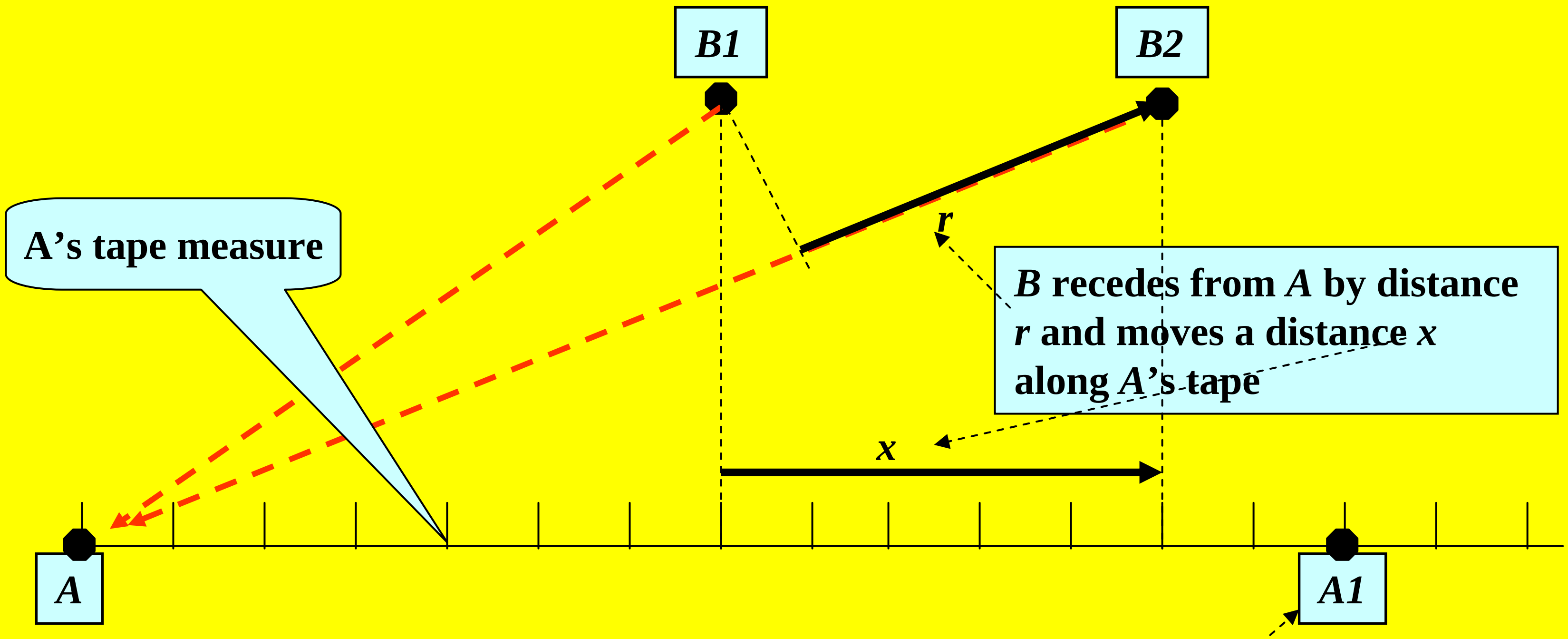
The meaning of V and V_r are illustrated on the next slide.

So there it is in a nutshell, but if you want know by what reasoning the above result was obtained and what the ramifications of this are, you will have to persevere.

So observed time intervals between events are not the same for everyone in spite of Newton's famous statement

'*time flows equably for everyone*'   Isaac Newton

ILLUSTRATING WHAT IS MEANT BY V AND V_r



But as you will see, we cannot say that the speed (V) of B relative to A is $V = x/t_A$ or that the speed that B is receding from A, (V_r) is $V_r = r/t_A$ as we would normally do

because if we did then another observer e.g. *A1* who is stationary relative to *A* and using the same tape as *A* would observe a different time interval (e.g. t_{A1}) and recessional distance (e.g. r_1) from that of *A* and would say that the speeds were $V = x/t_{A1}$ and $V_r = r_1/t_{A1}$ so no one using *A*'s tape would be able to agree what the values of V and V_r are.

These issues are dealt with on the next slide

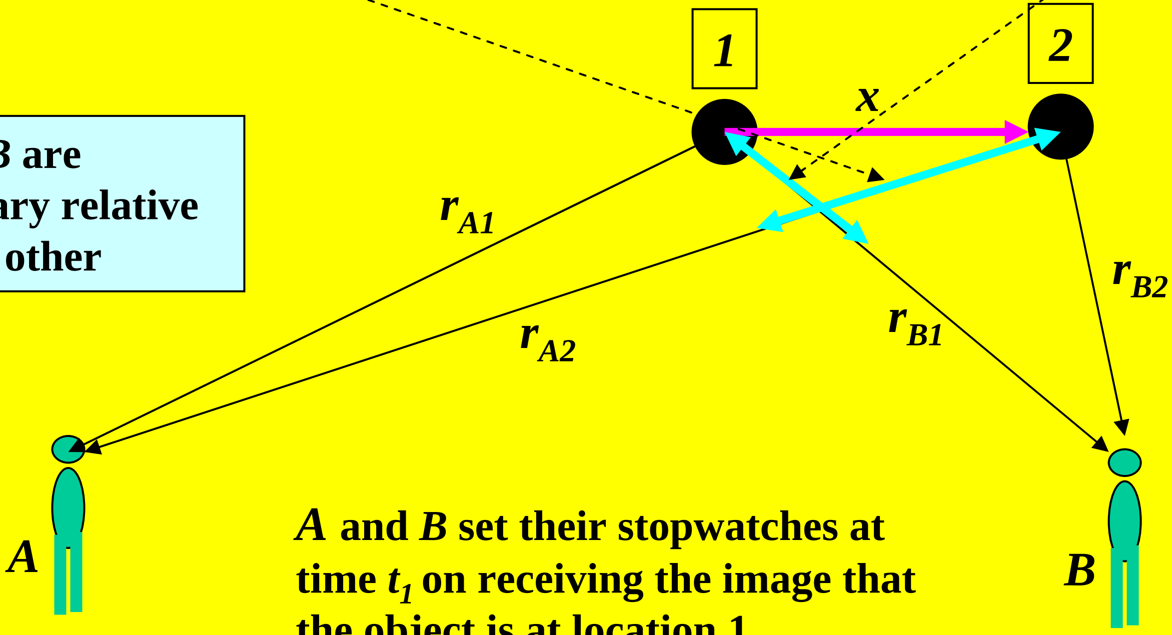
***A*'s second image had further to travel than the first image by the distance (r_{A2} - r_{A1}) so his observed time interval is increased by the time it takes for the second image to travel this extra distance i.e.**

$r_A \equiv (r_{A2} - r_{A1})/c$

So *A*'s observed time interval (t_{A2}-t_1) for the object to travel the distance *x* is greater than (t_{B2}-t_1) observed by *B*

***B*'s second image had less far to travel than the first image by the distance (r_{B1} - r_{B2}) so his observed time interval is decreased by $r_B \equiv (r_{B1} - r_{B2})/c$**

***A* and *B* are stationary relative to each other**



So ***A*** will say that the speed of the object is less than ***B*** because he **observed** it to take a longer time

***A* and *B* set their stopwatches at time t_1 on receiving the image that the object is at location 1.**

The object moves to location 2.

***A's* stopwatch reads time t_{A2} on receiving the image of the object at location 2**

***B's* stopwatch reads time t_{B2} on receiving the image of the object at location 2**

We cannot develop a theory which includes relationships involving the speed of objects if we cannot agree on what the speed of an object is. You (*A*) say it's one speed I (*B*) say its another.

We need to develop a new definition of speed whose value we can both agree upon. We both agree that the object moved the distance *x* but disagreed about how long it took and the disagreement arose out of consideration of the time it was taking for the images to reach us at speed *c*, because your two images and my two, had to travel different distances on account of where we happened to be standing.

Why don't we subtract from the times we observe on our stopwatches, the estimated times the images were taking to get to us. So when the object is a distance *r* away, we estimate that the image has taken the time *r*/*c* to reach us, we subtract this from the observed time *t* on our stopwatches, thus (*t* - *r*/*c*). We need a new symbol for this new time estimate, let's choose τ (taw) so

$$\tau \equiv t - r/c$$

NOTE that τ IS NOT A TIME VALUE WE CAN OBSERVE ON OUR STOPWATCH OR ANY CLOCK DEVICE.

It is the time we ESTIMATE that the image left the sender and involves OBSERVING

1. a TIME (*t*), 2. a DISTANCE (*r* , *i.e* SPACE)

and 3. KNOWING THE SPEED OF THE IMAGE (*c*).

It is a space and time variable.

Now we both agree about this time estimate which is known as 'event time' or 'relativistic time'.

$$\tau \equiv t - r/c$$

where

t **is the time interval we actually observe on our stopwatch**

and

r **is the the observed distance the object has receded from us on a measuring tape**

so we can define speed to be a value we both agree upon viz

$$V \equiv \frac{x}{\tau} = \frac{x}{t - r/c}$$

THE NEXT SERIES OF SLIDES SHOW HOW WE HAVE REASONED THAT

$$\frac{t_A}{t_B} = \frac{1+\left(V_r/c\right)}{\sqrt{1-\left(V/c\right)^2}}$$

We shall be using the words 'sender' (or 'emitter') of signals and 'receiver' of those signals.

The words 'sender' or 'emitter' do not imply that some transmitting device is necessarily being used to send signals. We see objects usually because they are reflecting light to us so they are sending us light signals (images) although they are not actively emitting light they are still the sender of the image. We are the receiver of these signals (images) simply by viewing them. So these terms need not imply any physical transmitting and receiving equipment.

The word signal here just indicates that some information has been transmitted by electromagnetic waves (e.g. images, radio etc).

We are tagging *B* as the sender of the signals and *A* as the receiver of the signals.

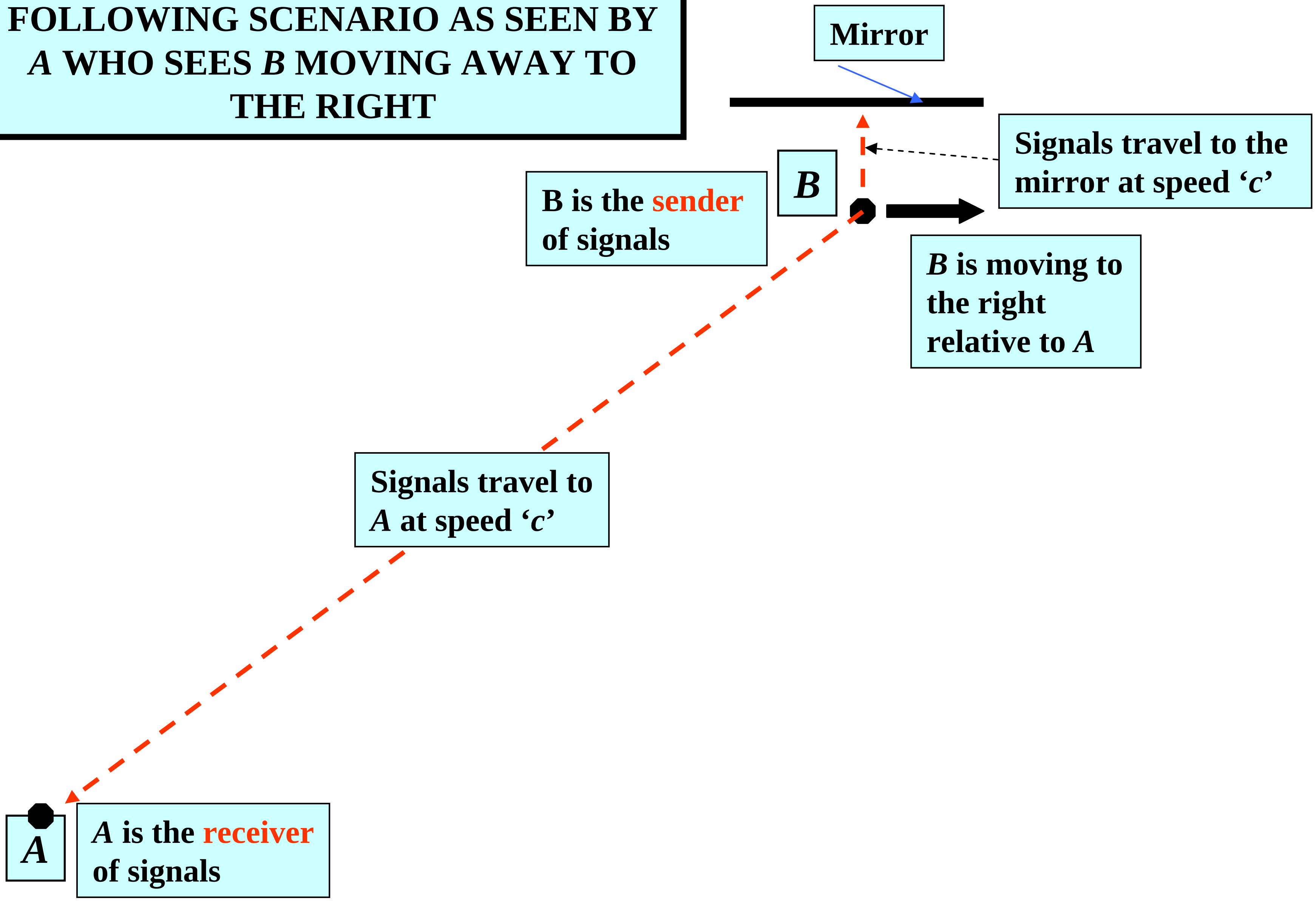
Speeds V and V_r are the speeds that *A* observes *B* to be moving.

ONLY IF YOU WANT TO KNOW

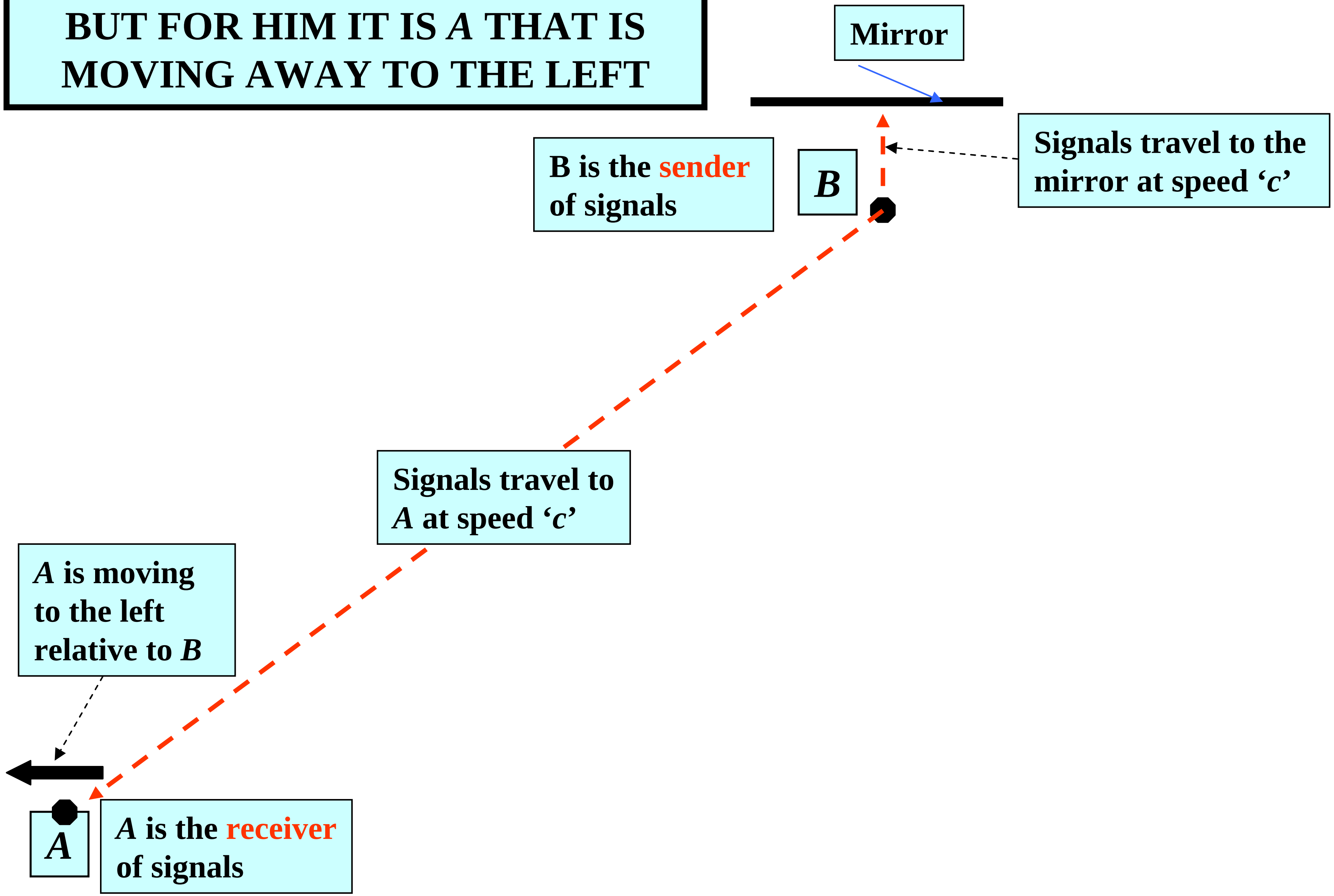
Some References to the equation $\frac{dt_A}{dt_B} = \frac{1+\beta}{\sqrt{1-\beta^2}}$

Born, M.	Einstein's Theory of Relativity.	p300
Broullouin, L.	Relativity Re-examined.	p61
Ellis, G. & Williams R. M.	Flat and Curved Space–times.	p62
French, A. P.	Special Relativity.	p137
Harrison	Cosmology	p306
Poole, C	Handbook of Physics	Ch7
Stevenson and Kilmister	Special Relativity for Physicists.	p17

WE ARE ABOUT TO CONSIDER THE FOLLOWING SCENARIO AS SEEN BY *A* WHO SEES *B* MOVING AWAY TO THE RIGHT

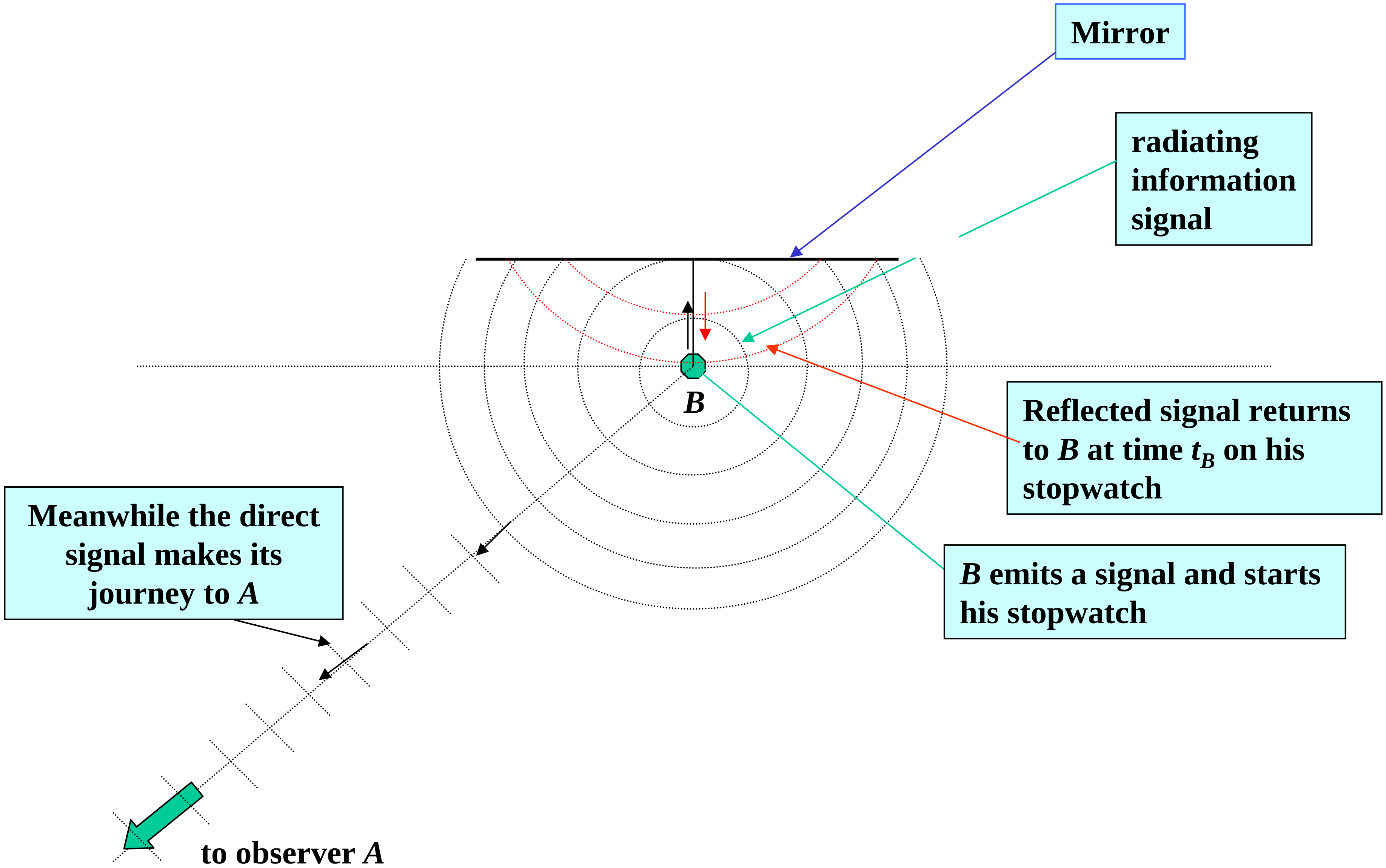


B SEES A SIMILAR SCENARIO BUT FOR HIM IT IS *A* THAT IS MOVING AWAY TO THE LEFT

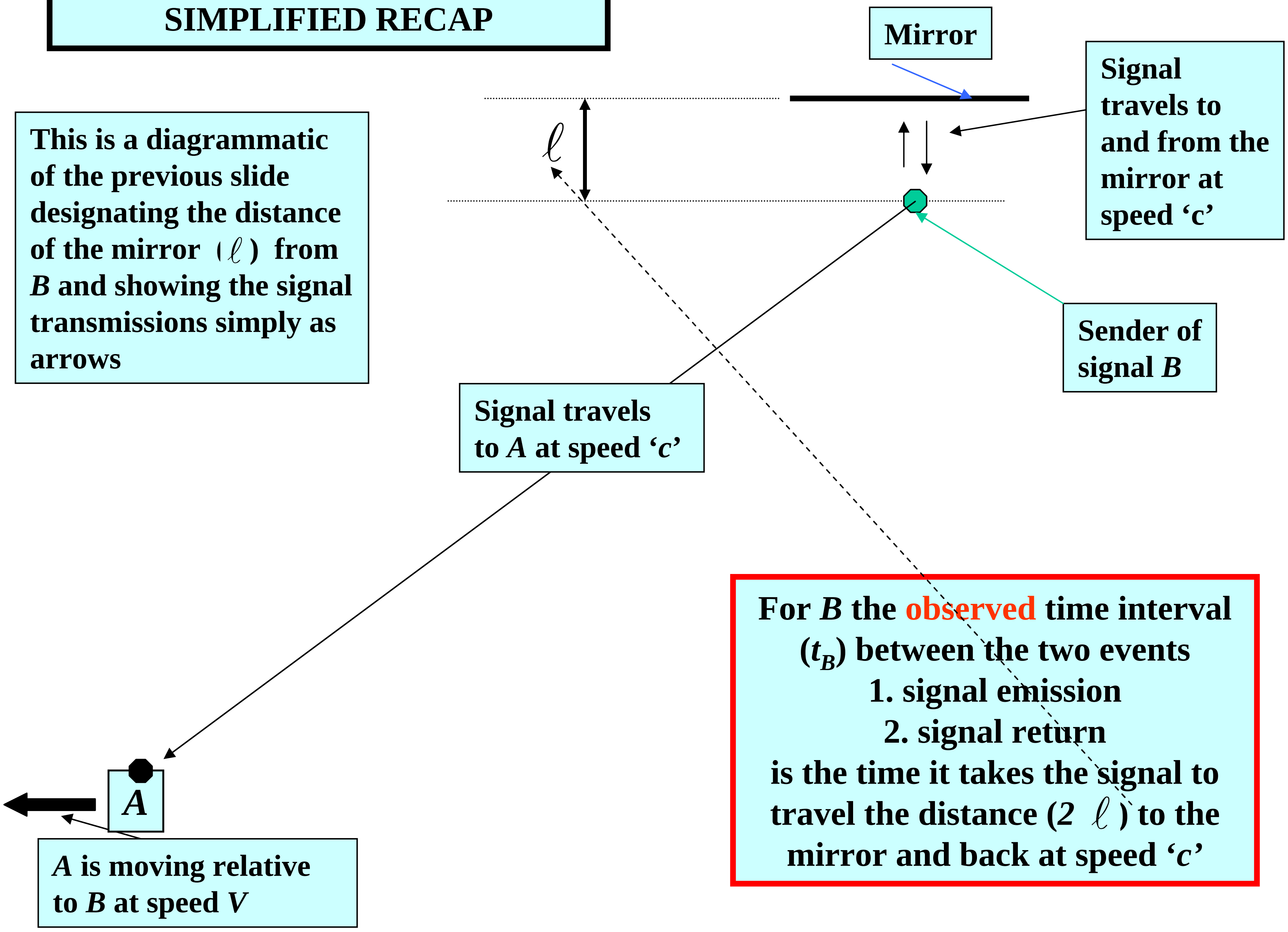


Scenario 1a

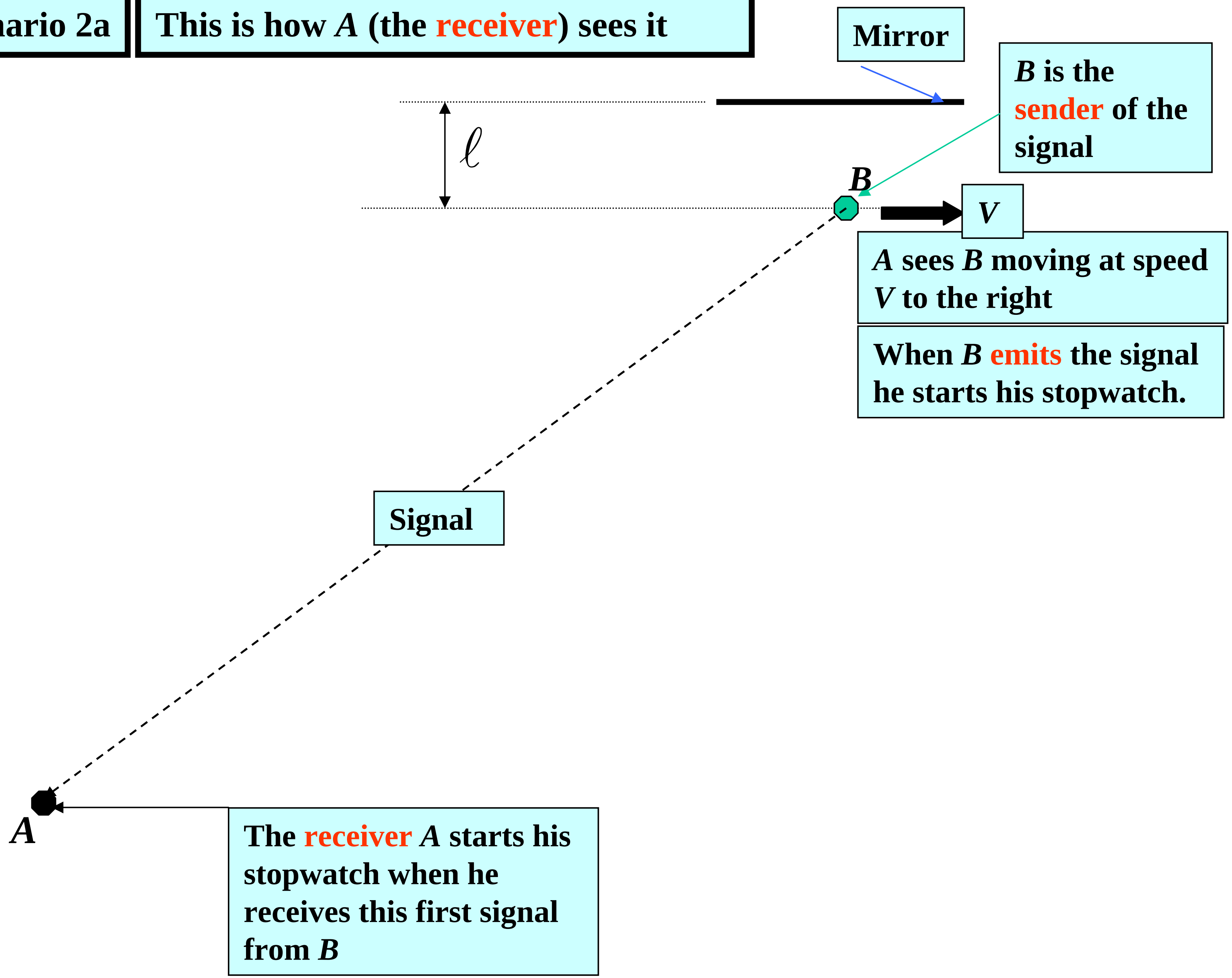
This is how *B* (the sender) sees this scenario locally.



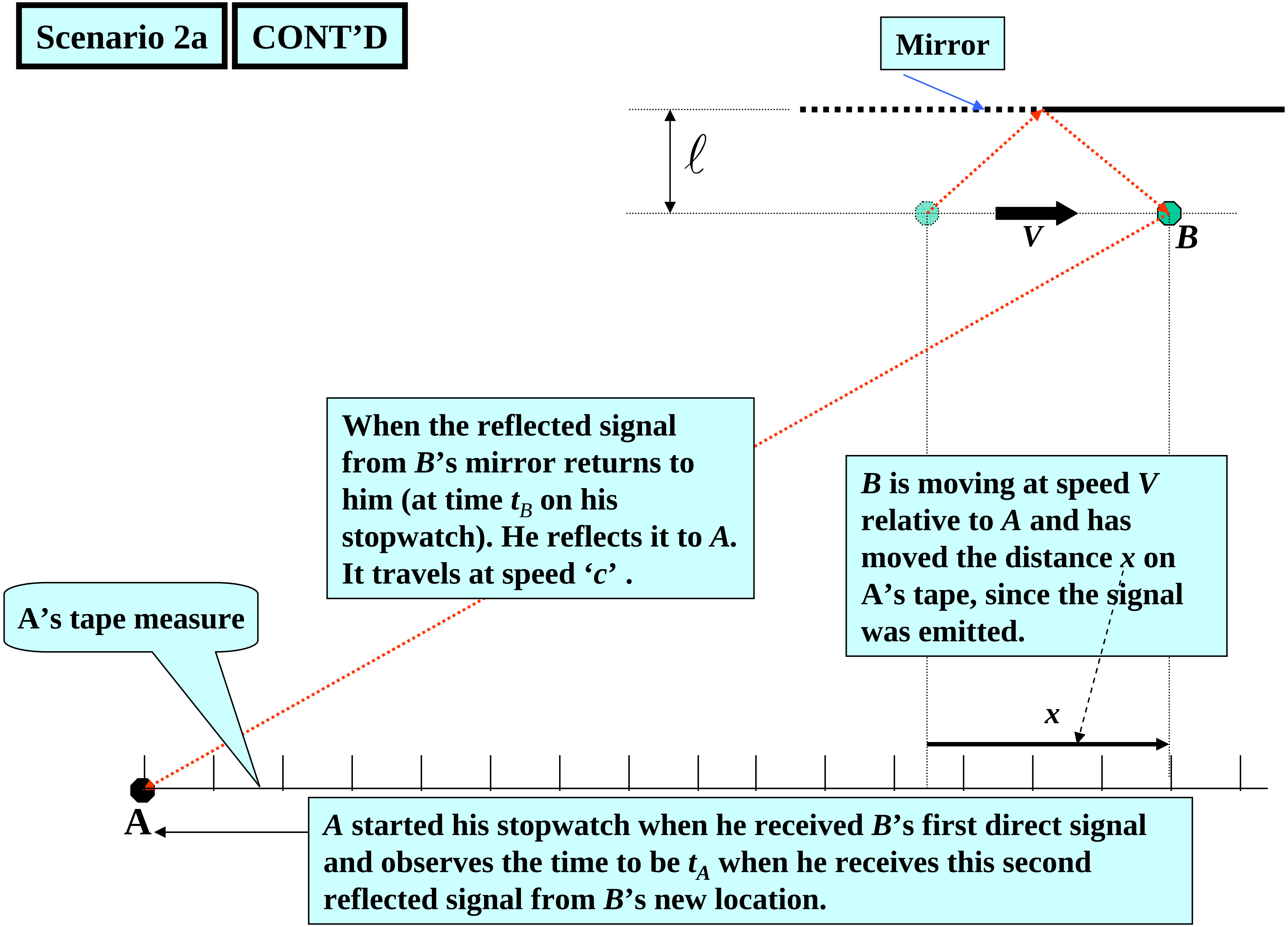
SIMPLIFIED RECAP



Scenario 2a | This is how *A* (the receiver) sees it



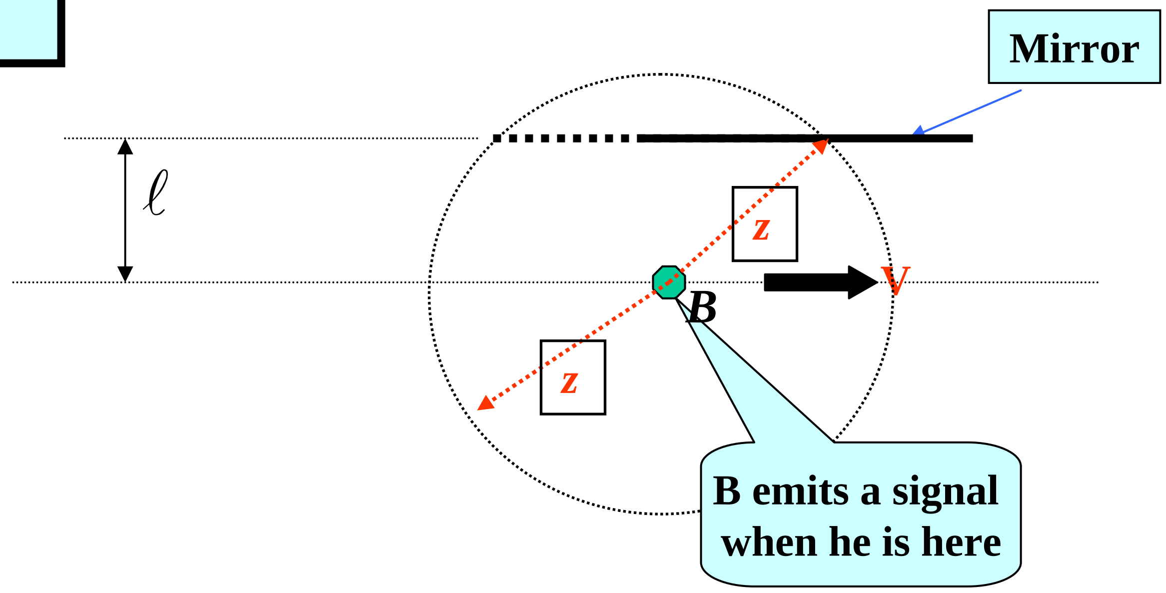
Scenario 2a CONT'D

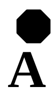


Scenario 2a

CONT'D

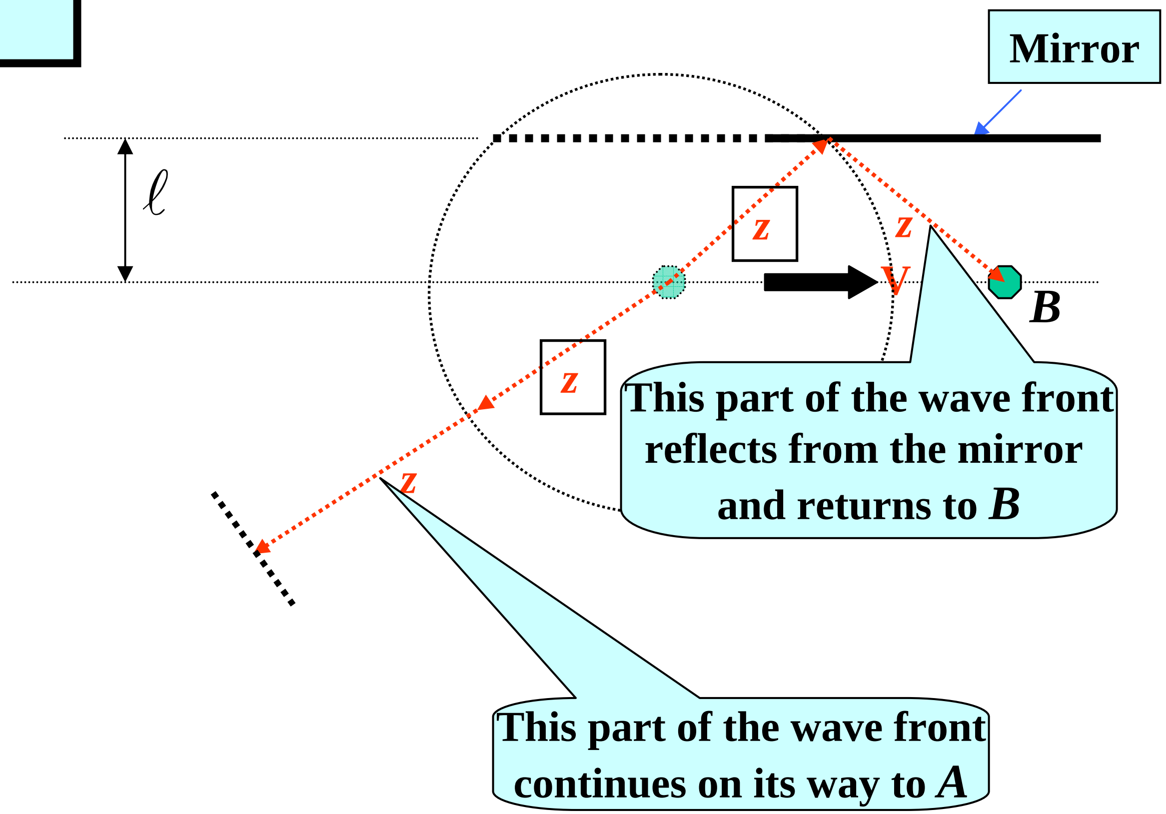
We now consider the distances the direct and the reflected signals move.





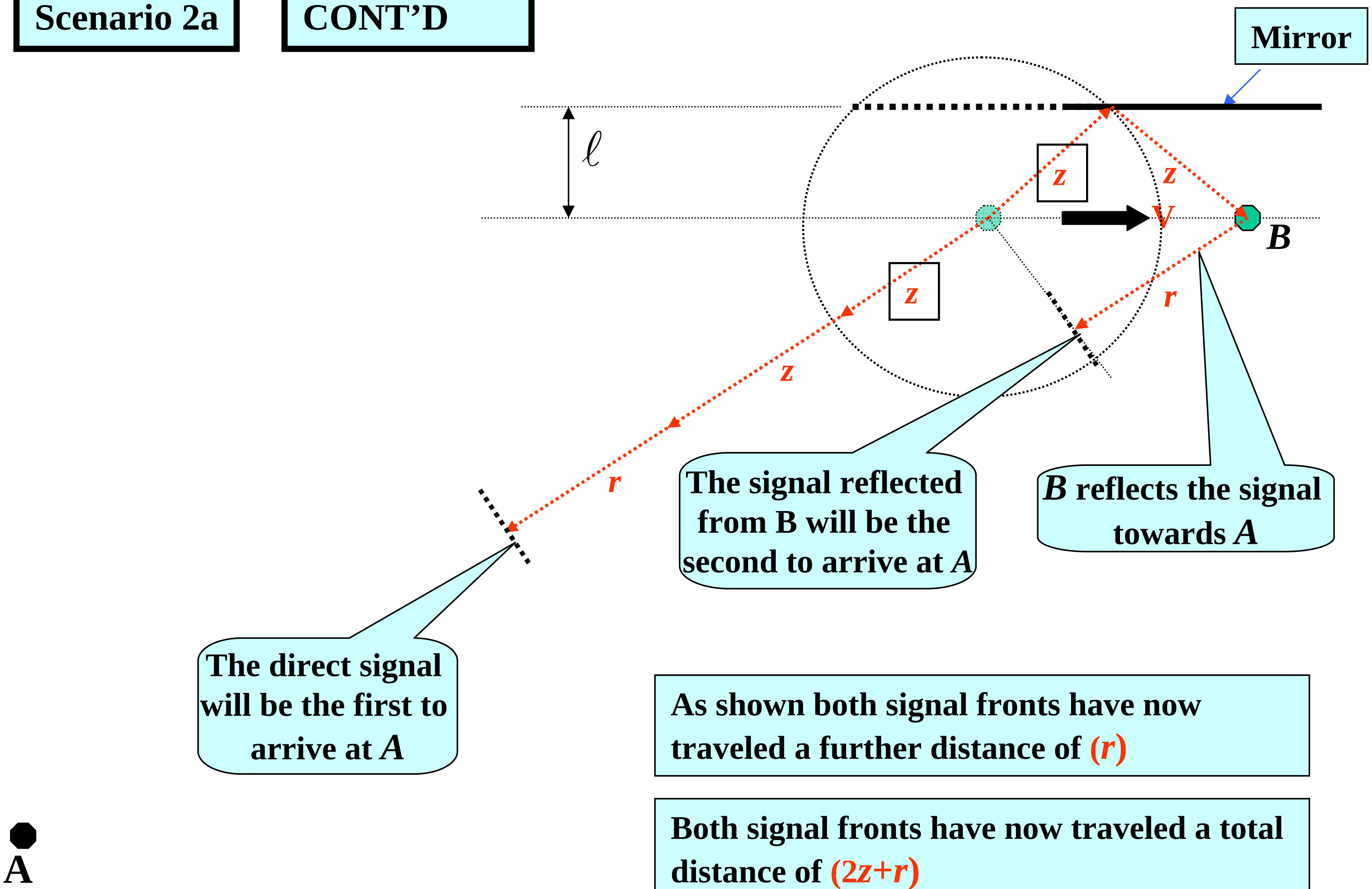
Scenario 2a

CONT’D



Both wave fronts have now traveled the same total distance 2*Z*.

Scenario 2a **CONT'D**



As shown both signal fronts have now traveled a further distance of (r)

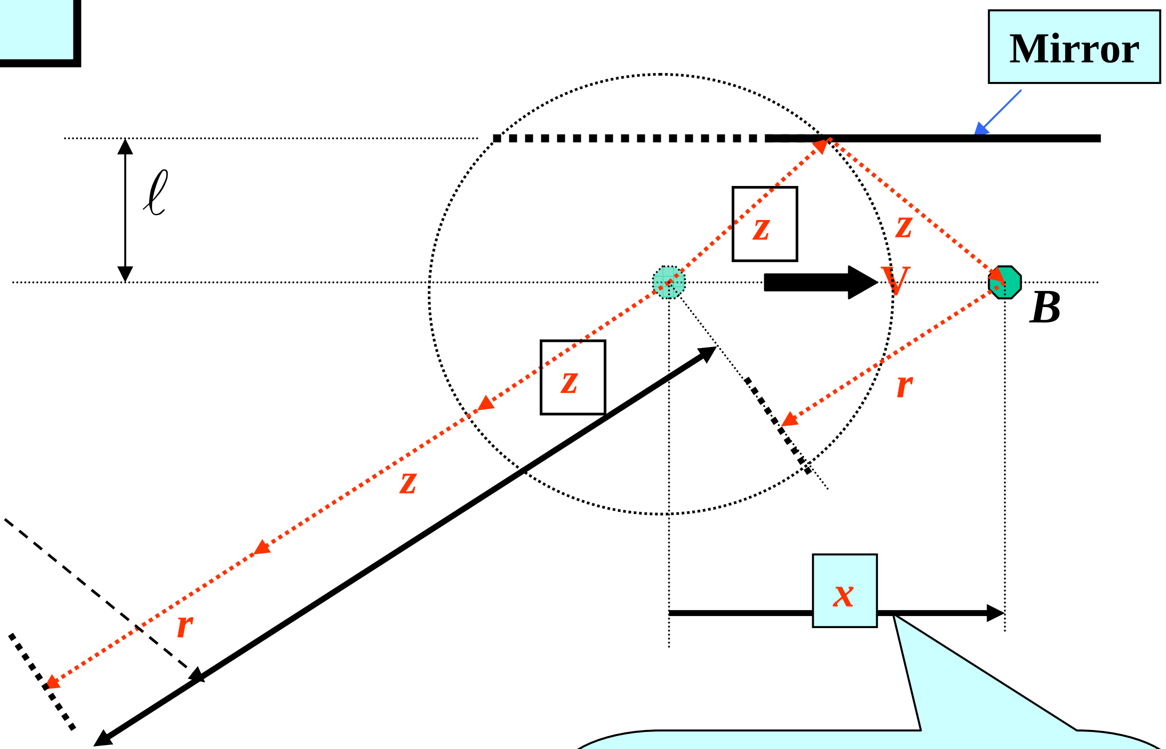
Both signal fronts have now traveled a total distance of ($2z+r$)

Scenario 2a **CONT'D**

***B*'s reflected signal travels behind the direct signal by the distance**

z+z+r = 2z+r

so the reflected signal arrives at *A* after the first signal by the time it takes the second signal to travel this distance *2z+r* at speed *c*.



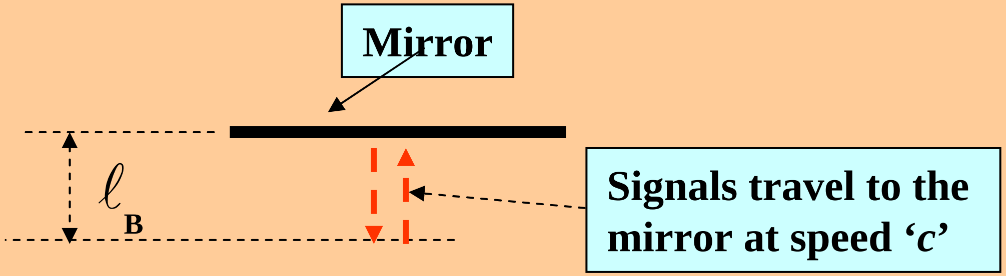
A* observes *B* to have moved a distance *x* and receded from him by the distance *r* in the time that the signals have traveled the distance 2*z*+*r

A

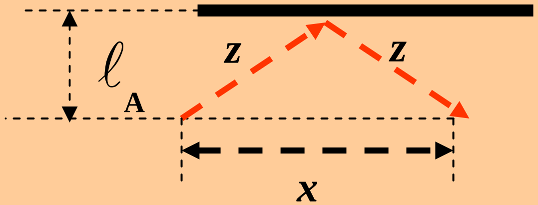
For *A* the time interval (t_A) between the two events (emission of the signal and its return to *B*) is the time it takes the second signal to travel the distance (*2z+r*) at speed *c*.

ONLY IF YOU WANT TO KNOW

It should not go unobserved (for future considerations) that when *B* observes the mirror to be a lateral distance ℓ_B from him, we have tacitly assumed that *A* will observe the lateral distance ℓ_A of the mirror from him (i.e B) to be the same



Because we said that *B* observes the time to travel to the mirror and back was a distance of $2\ell_B$ we then said that from *A*'s point of view the signal traveled a distance of 2z where by Pythagoras



$$(x/2)^2 + \ell_A^2 = z^2$$

But we didn't distinguish between them, we just used ℓ for both as if it was the same for both of them as if $\ell_A = \ell_B$! This tacit assumption has never been actually verified experimentally.

WE MAKE THE FOLLOWING FOUR DEDUCTIONS FROM WHAT WE HAVE JUST OBSERVED

Deduction No 1

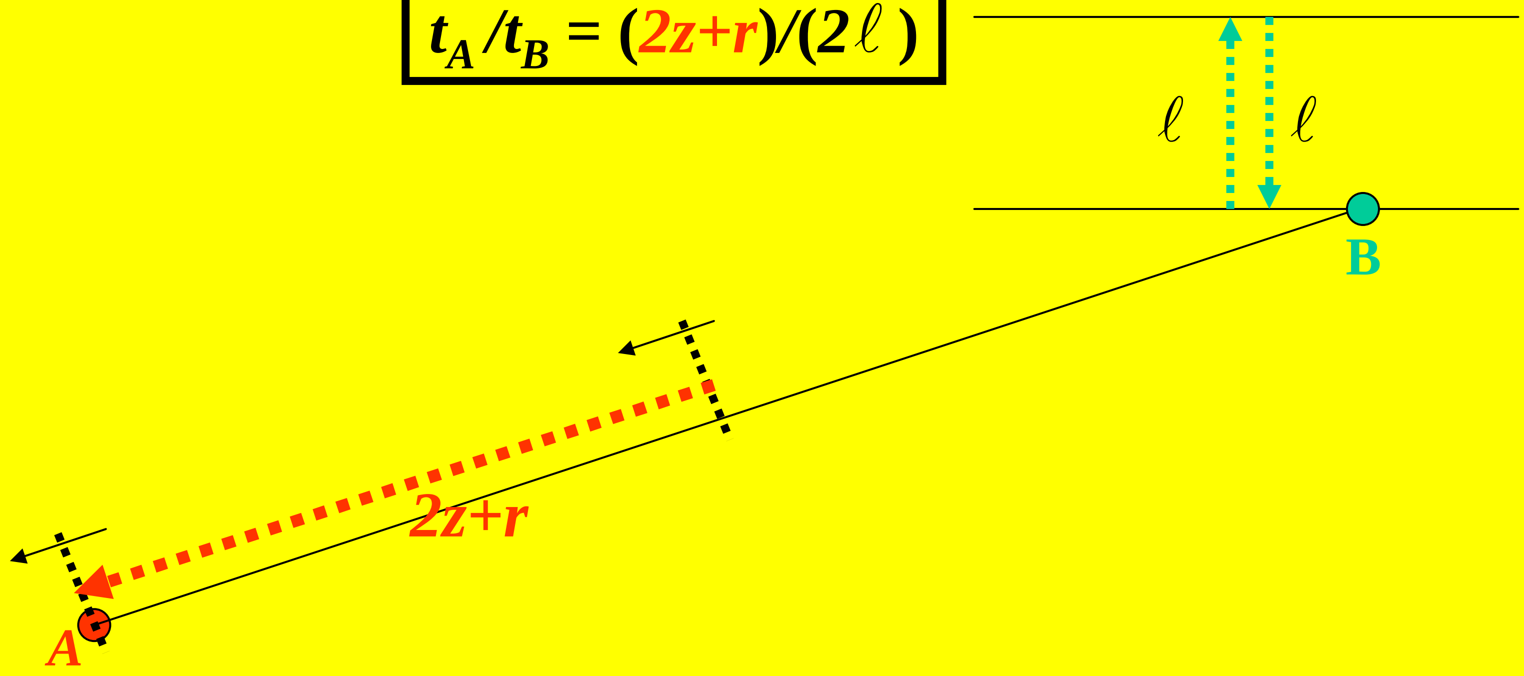
The ratio of the time interval for A (t_A) to that for B (t_B) between the two events that occur at B (i.e. the emission from and return of the signal to B) is then the same as the ratio of the distances the signals have to travel for A and for B (since the signals travel at the same speed 'c' for both of them).

For A this is the time for the signal to travel the distance ($2z+r$)

For B this is the time for the signal to travel the distance (2ℓ)

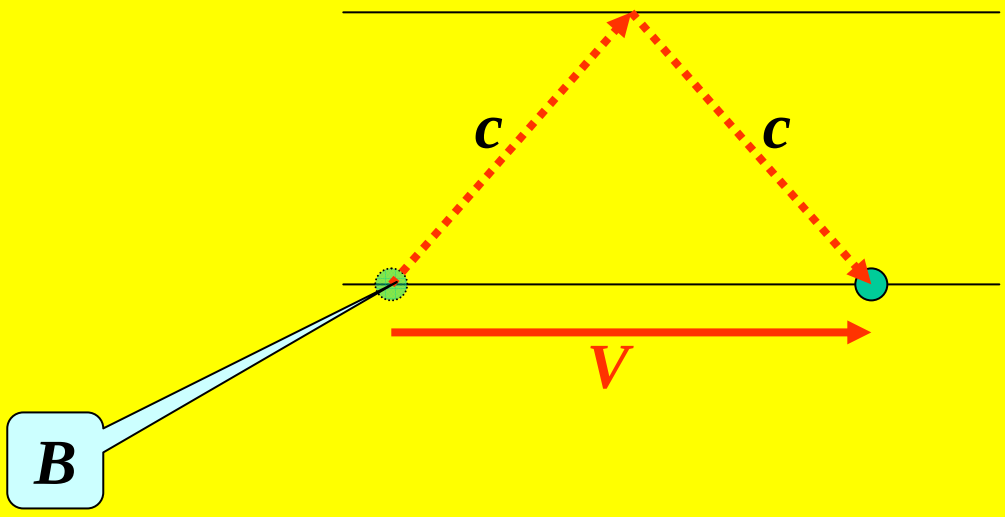
so the ratio is

$$t_A/t_B = (2z+r)/(2\ell)$$

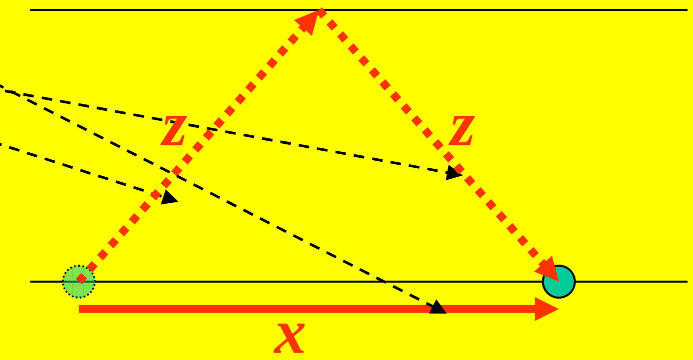


Deduction No 2 from what *A* observes

The ratio of *B*'s speed (*V*)
to the speed of the signal (*c*)



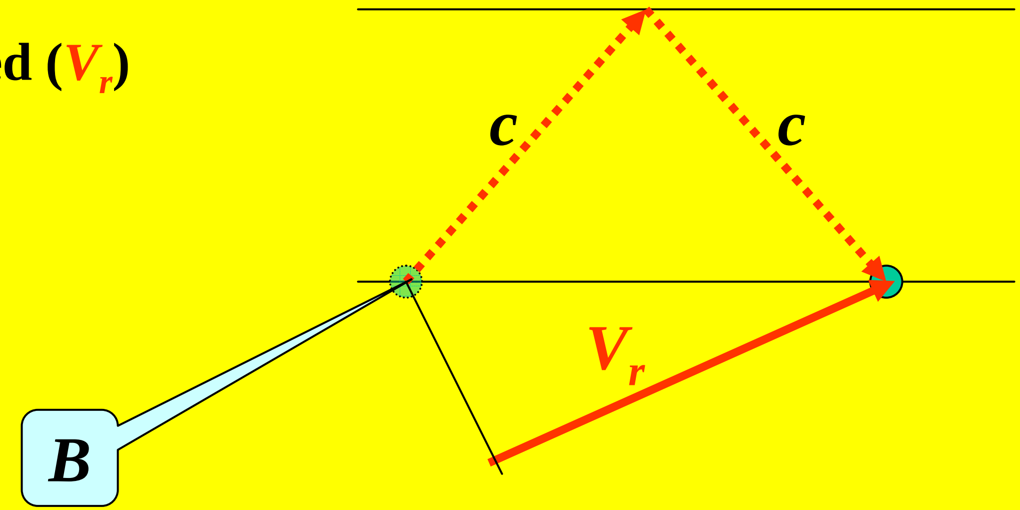
is the ratio of the distance *x* that *B* moves along *A*'s tape to the distance 2*z* that *A* concludes that the signal moves in the same time interval



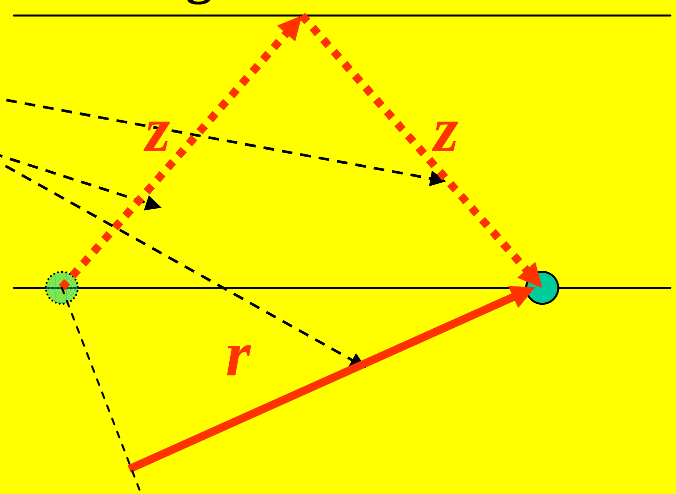
so $V/c = x/(2z)$

Deduction No 3 from what *A* observes

The ratio of *B*'s recessional speed (V_r)
to the speed of the signal (c)



is the ratio of the distance r that *B* moves away
from *A* to the distance $2z$ that *A* concludes that the signal moves
in the same time

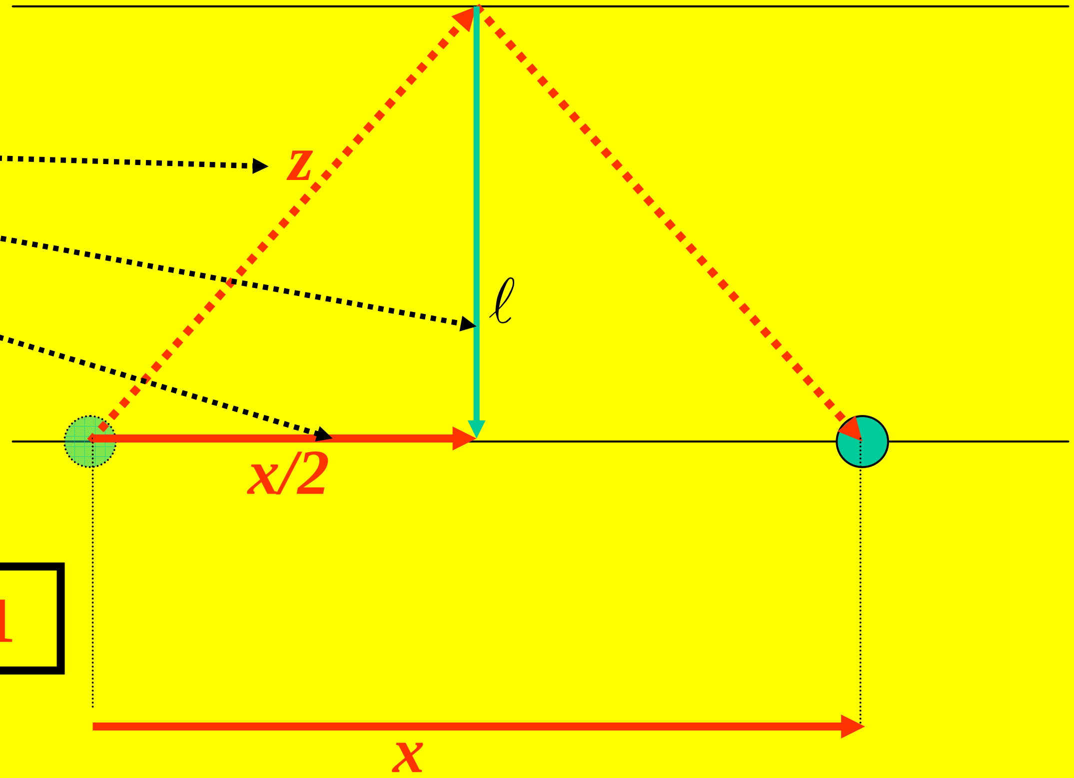


so $V_r/c = r/(2z)$

Deduction No 4 from what *A* observes

Pythagoras theorem tells us that

$$(x/2)^2 + \ell^2 = z^2$$



which may be expressed as ratios simply by dividing by z^2

$$(x/2z)^2 + (\ell/z)^2 = (z/z)^2 = 1$$

Assembling our four Deductions

Remember
A is the receiver
B is the sender

$$t_A / t_B = (2z+r)/(2\ell)$$

$$V/c = x/(2z)$$

$$V_r/c = r/(2z)$$

$$(x/2z)^2 + (\ell/z)^2 = 1$$

⇩

We can derive from the above (with a little tedious algebra) the following result

⇩

$$\frac{t_A}{t_B} = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}}$$

ONLY IF YOU WANT TO KNOW

By dividing by τ_A we can re-arrange our previous result $\tau_A \equiv t_A - r_A/c$

to read $t_A = \tau_A(1+V_r)$

and observing that $\tau_B \equiv t_B - r_B/c = \tau_B \equiv t_B - 0/c = t_B$

If we substitute these values for t_A and t_B into our kernel result

$$\frac{t_A}{t_B} = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}}$$

it reads as

$$\frac{\tau_A}{\tau_B} = \frac{1}{\sqrt{1-(V/c)^2}}$$

Which is the way you will see it in most text books although they won't usually tell you that τ_A and τ_B are not times that can be observed on any clock device.

ONLY IF YOU WANT TO KNOW

If we are only interested in motions in line-of-sight (i.e. direct approach or recession) then $V = V_r$ and our previous result can be written

$$\frac{t_A}{t_B} = \frac{\nu_B}{\nu_A} = \frac{1+(V/c)}{\sqrt{1-(V/c)^2}} = \sqrt{\frac{1+(V/c)}{1-(V/c)}}$$

where frequency ν is simply the the reciprocal of time interval i.e. $\nu = 1/t$ e.g. if events occur at intervals of 1/2 s then their frequency is 2 /s (twice per second)

so from the above $$\nu_A = \nu_B \sqrt{\frac{1-(V/c)}{1+(V/c)}}$$

Below is a cutting from Einstein's June 1905 paper 'ON THE ELECTRODYNAMICS OF MOVING BODIES' (p16 of the English translation) which subsequently appeared in the book 'THE PRINCIPLE OF RELATIVITY' (Methuen 1923)

This is Doppler's principle for any velocities whatever. When $\phi = 0$ the equation assumes the perspicuous form

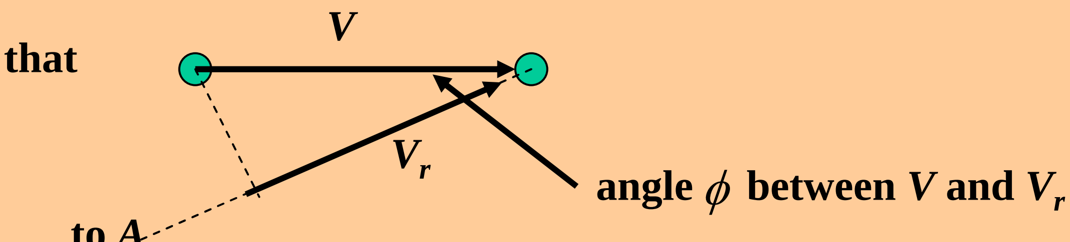
$$\nu' = \nu\sqrt{\frac{1 - v/c}{1 + v/c}}.$$

We see that, in contrast with the customary view, when $v = -c, \nu' = \infty$.

ϕ = 0 means line-of-sight

ONLY IF YOU WANT TO KNOW

Notice from earlier diagram that



So $V_r = V \cos\phi$

So since frequency ν is the reciprocal of time interval t then

$$\frac{t_A}{t_B} = \frac{\nu_B}{\nu_A} = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}} = \frac{1+\cos\phi\times(V/c)}{\sqrt{1-(V/c)^2}}$$

and so

$$\nu_A = \nu_B \frac{\sqrt{1-(V/c)^2}}{1+\cos\phi\times(V/c)}$$

But Einstein says

$$\nu_A = \nu_B \frac{1-\cos\phi\times(V/c)}{\sqrt{1-(V/c)^2}}$$

But when the emitter moves transverse to the line of sight time there is no Doppler effect because there is no recessional motion, only time dilatation, which increases the observers time interval and must therefore decrease the observers frequency as shown on the next slide but one.

ONLY IF YOU WANT TO KNOW

Below is a cutting from the English translation of Einstein's paper where he uses $\nu = \nu_A$ and $\nu = \nu_B$. The same equations appear in the original German version.

On the Electrodynamics of Moving Bodies file:///R:/Einstein.ht

with the velocity of the observer referred to a system of co-ordinates which is at rest relatively to the source of light, the frequency ν' of the light perceived by the observer is given by the equation

$$\nu' = \nu \frac{1 - \cos\phi \cdot v/c}{\sqrt{1 - v^2/c^2}}.$$

This is Doppler's principle for any velocities whatever. When $\phi = 0$ the equation assumes the perspicuous form

$$\nu' = \nu \sqrt{\frac{1 - v/c}{1 + v/c}}.$$

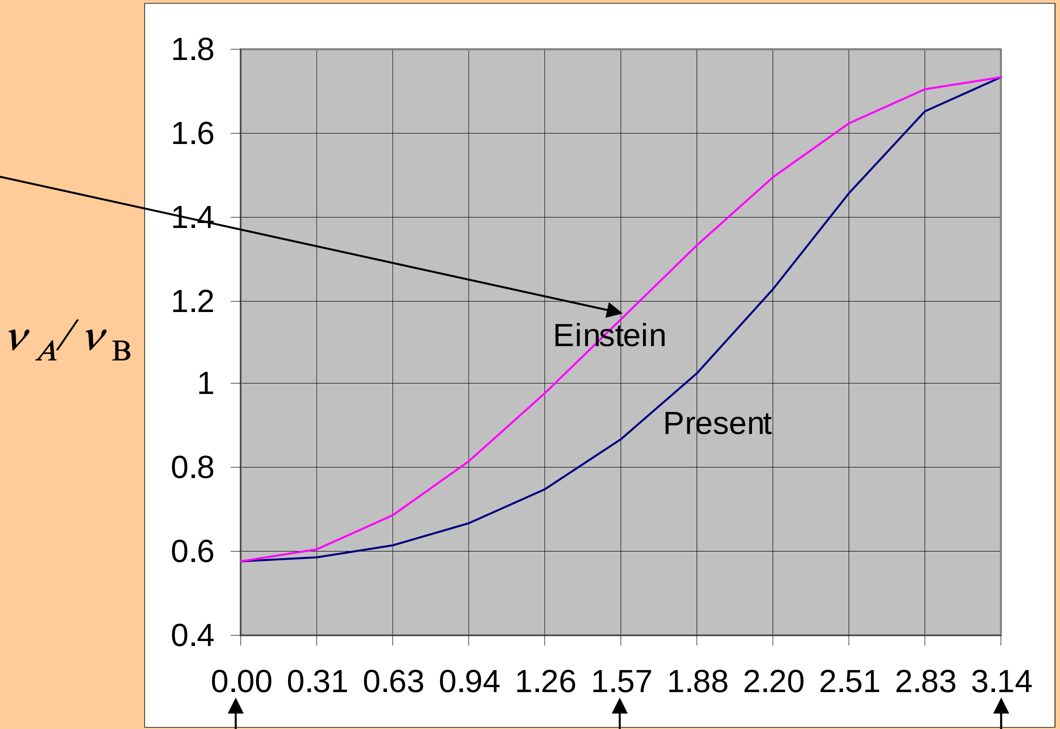
We see that, in contrast with the customary view, when $v = -c$, $\nu' = \infty$.

He states contrary to the customary view (i.e. the acoustic case where there is no time dilatation) that when the emitter is approaching at the speed of the signal ($V = -c$) the receiver frequency is infinite $\nu' = \infty$ but this is true also in the customary (acoustic) case as the signals travel with the emitter and all arrive together. Clearly Einstein has got this wrong. Comparisons are shown on the next slide.

ONLY IF YOU WANT TO KNOW

Comparison of frequency ratio of observer/emitter for the case of β = 0.5, as seen by the observer at different angles from direct approach at ϕ = 3.14 rad to direct recession at ϕ = 0 rad, as argued in the current presentation vs Einstein.

Einstein is saying that the receiver's frequency is greater than the emitter's when he is crossing the receiver's line of sight i.e. the receiver's time intervals are shorter than the emitter's. This would constitute negative time dilatation !!



Direct recession

Transverse to the line of sight

Direct approach

Just a final word of caution about this kernel of special relativity

$$\frac{t_A}{t_B} = \frac{1 + (V_r/c)}{\sqrt{1 - (V/c)^2}}$$

Recall that *A* is observing events at *B*'s location so signals of the events have to travel zero distance to reach *B*, hence $r_B = 0$ and so

$$\tau_B \equiv t_B - r_B/c = t_B - 0/c = t_B$$

for *B* event time and observed time are the same thing

Also since $$\tau_A = \frac{t_A}{1 + V_r/c}$$

You will most often see the above written as

$$\frac{\tau_A}{\tau_B} = \frac{1}{\sqrt{1 - (V/c)^2}}$$

but you may not be aware that τ is not some time that can be observed on a stopwatch (or any clock device), it refers to the estimated event time.

THE SPECIAL THEORY OF RELATIVITY

Henri Poincaré was known as the father of relativity and coined the term 'theory of relativity' in his 1904 paper. The term does not appear in either of Einstein's 1905 papers which were titled 'On the Electrodynamics of Moving Bodies' and ' Does the Inertia of a Body Depend on its Energy Content', respectively. These two papers are generally known as Einstein's theory of relativity but the substance of them had already appeared in Poincaré's 1904 paper albeit by a more mathematical approach. Throughout his life Einstein denied having seen Poincaré's 1904 paper.

Relativity here implies that if any two observers moving at constant velocity one relative to the other, conduct identical experiments and compare their results, they will not observe any difference between them. This is based on the fact that there is no apparent reason why nature would show any preference to either of them and is borne out by observation.

Prior to the 19th century before the discovery of electro-magnetic phenomena, when only mechanical effects were understood, the theory is known as Newtonian-Galilean Relativity. Like Poincaré and Einstein's relativity it embodied the idea that there is no such thing as absolute rest (any motion is simply of one thing relative to another) but unlike Einstein's relativity it assumed that time is absolute and the same for everyone under all situations.

'*time flows equably for everyone'* Isaac Newton

WELL AS WE HAVE JUST SEEN THIS IS NOT SO.

The theory is Special because it excludes consideration of any effects which gravitation might have. The later theory which includes gravitational effects is known as the General theory of relativity.

NEXT, A BRIEF REVIEW OF SOME HISTORICAL BACKGROUND

After

(1785) Charles Coulomb discovers the force between spaced static electric charges

(1825) André Ampére discovers the force between spaced static conductors carrying electric currents, subsequently formulated by Jean Biot and Felix Savart.

(1831) Michael Faraday discovers that magnets moving with respect to electric conductors induce electric currents. This introduces time into relationships because it now involves motion.

(1865) James Clerk Maxwell was able to develop the following equations describing electro-magnetic field disturbances in space (*x*) and time (*t*) from the endeavours of the above workers

$$\frac{1}{(\varepsilon\mu)}\frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{1}{(\varepsilon\mu)}\frac{\partial^2 H}{\partial x^2} - \frac{\partial^2 H}{\partial t^2} = 0$$

where *E* and *H* are the electric and magnetic field strengths at distance x from the cause, and ε and μ are the electrical permittivity and magnetic permeability respectively, of the medium separating the electrical cause and the observed effect.

This equation form

$$c^2 \frac{\partial^2 X}{\partial x^2} - \frac{\partial^2 X}{\partial t^2} = 0$$

was well known to describe waves moving with speed *c* [as in acoustics where *X* can be the pressure (*p*) or density (ρ) of the medium and $c^2 = dp/d\rho$ so $c = \sqrt{dp/d\rho}$

So James Clerk Maxwell's (1865) equations

$$\frac{1}{(\varepsilon\mu)} \frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{1}{(\varepsilon\mu)} \frac{\partial^2 H}{\partial x^2} - \frac{\partial^2 H}{\partial t^2} = 0$$

were clearly describing electro-magnetic waves traveling at a speed $c = \frac{1}{\sqrt{\varepsilon\mu}}$

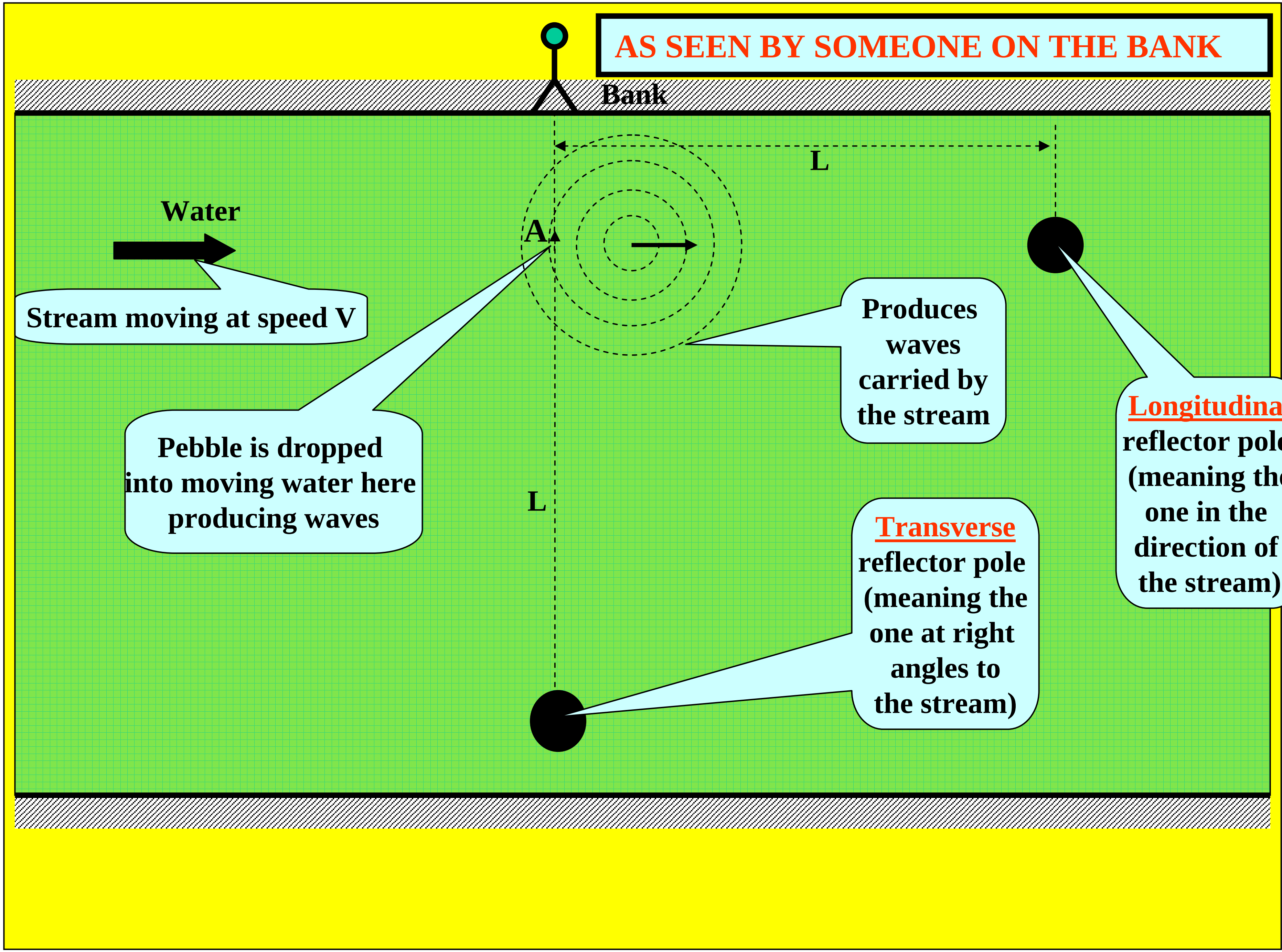
(1885-1889) Heinrich Hertz established that radio and then light waves were electro-magnetic waves. The above equations became known as the Maxwell-Hertz equations and that electrical disturbances would cause such waves in the electric and magnetic fields and that the speed of such waves (disturbances) would then be *c* given above.

When empty space (vacuum) separates the cause and the observed effect, these properties have been determined by static bench top experiments in SI units in vaccuo as

$\varepsilon = 8.854 \times 10^{-12}$ *and* $\mu = 1.25 \times 10^{-6}$ **then** $c = 2.997 \times 10^{8}\ m/s$

Much effort was then directed to detecting the motion of the ether medium pervading even a vacuum, on which these waves travel

THE NEXT SLIDES CONSIDER SOME OF THE BASIC THINKING WHICH FOLLOWED REGARDING WAVES AND WAVE REFLECTION


AS SEEN BY SOMEONE ON THE BANK
Bank
L
Water
A
Stream moving at speed V
Produces waves carried by the stream
Pebble is dropped into moving water here producing waves
Longitudinal reflector pole (meaning the one in the direction of the stream)
L
Transverse reflector pole (meaning the one at right angles to the stream)

The waves travel to the reflectors and back to A.

The distance to the longitudinal and transverse reflectors is the same $L_l = L_t$

The waves travel at speed *c* on still water and the speed of the stream is *V*.

The time taken for the waves travel to the longitudinal reflector and back to A is = $\frac{2L}{c} \times \gamma^2$

The time take for the waves travel to the transverse reflector and back to A is = $\frac{2L}{c} \times \gamma$

Where we have replaced $\frac{1}{\sqrt{1-(V/c)^2}}$ **by** γ **to reduce visual clutter.**

As you can see these two times are not the same

The difference between them is $\frac{2L_l}{c} \times \gamma^2 - \frac{2L_t}{c} \times \gamma$

Time for waves to travel to the longitudinal reflector and back

Time for waves to travel to the transverse reflector and back

By taking the difference between these two times we can determine the value of *V/c* (i.e. the ratio of the speed of the stream '*V*' relative to the speed of the waves in still water '*c*')

So replacing $\frac{1}{\sqrt{1-(V/c)^2}}$ **by** γ **to reduce clutter the previous difference can be written as**

$$\frac{2L_l}{c} \times \gamma^2 - \frac{2L_t}{c} \times \gamma$$

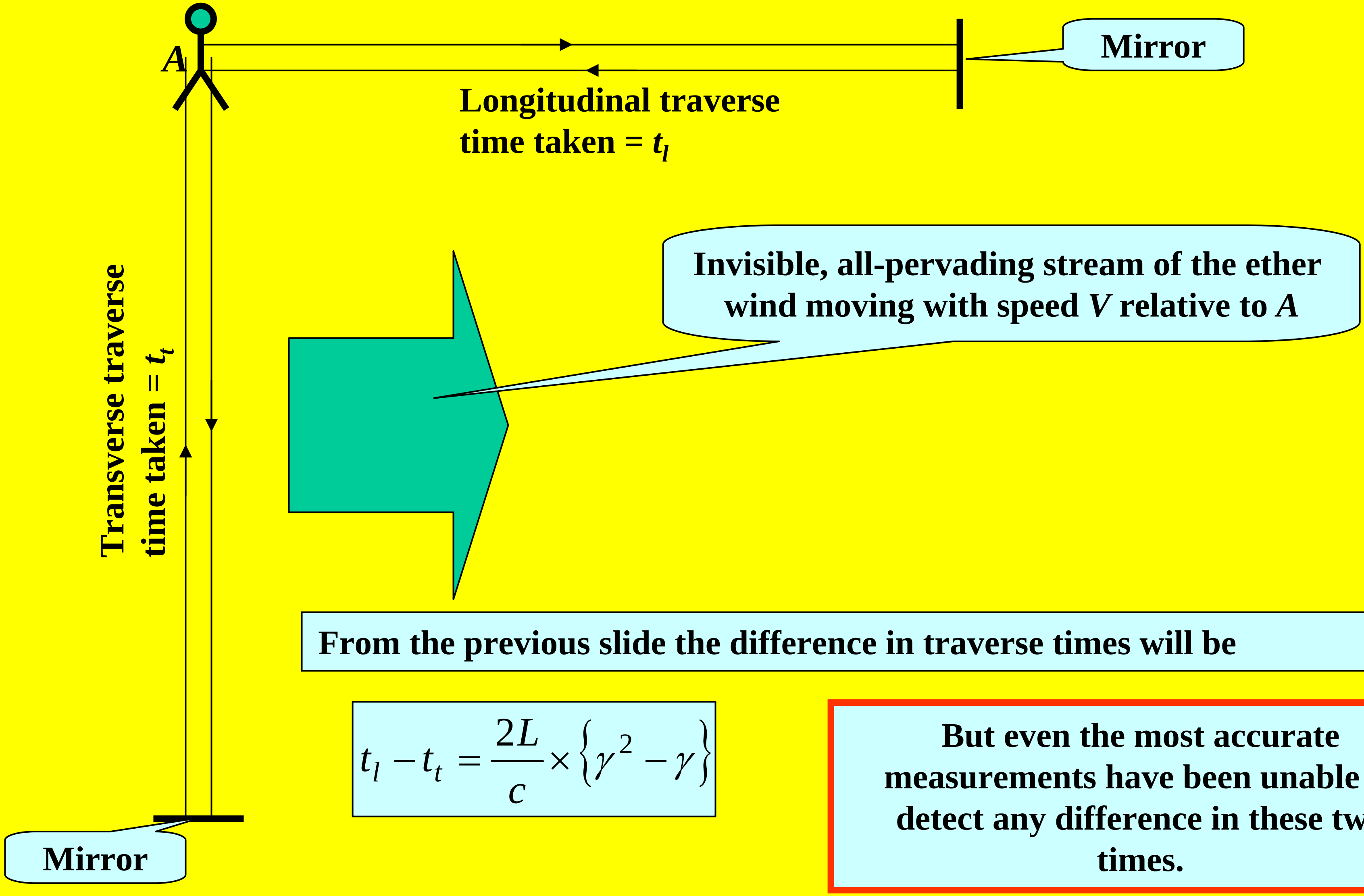
Where the distance to the two reflectors is the same $L_l = L_t$

Time for waves to travel to the longitudinal reflector and back

Time for waves to travel to the transverse reflector and back

Notice that if the distance to the longitudinal reflector L_l **, was magically and invisibly reduced to** $\frac{L_l}{\gamma}$ **then the two expressions would be the same and the time difference would become zero.** **<u>REMEMBER THIS</u>.**

Transpose the previous scenario to the case where the ether replaces the water, the reflectors are plane mirrors and the waves are light waves.



From the previous slide the difference in traverse times will be

$$t_l - t_t = \frac{2L}{c} \times \left\{ \gamma^2 - \gamma \right\}$$

But even the most accurate measurements have been unable to detect any difference in these two times.

Albert Michelson carried out this experiment using reflecting mirrors and light waves at Berlin in 1881.

He and Edward Morley refined the apparatus to provide a very high accuracy at Ohio in 1887. Even measuring to a few parts in a million, no time difference could be detected.

What's the explanation ?

In 1892 George Fitzgerald working at Trinity College Dublin came up with the hypothesis that all matter contracts in length in the direction of the ether wind by the factor $\frac{1}{\gamma} \leq 1$ **so that the true distance to the longitudinal mirror in the direction of the ether wind becomes**

not L **but** $\left(\frac{L}{\gamma}\right)$ **so the difference between the two times becomes**

$$\left(\frac{L}{\gamma}\right) \times \gamma^2 - L \times \gamma = \boxed{0}$$

in agreement with observations

In 1895 Hendrik Lorentz working at Leiden claims to have come up independently with the same idea(?) but developed the idea and its implications much further than Fitzgerald.

There were serious objections to this length contraction idea. The contraction could never be observed because any device used to measure it would itself contract by the same amount.

From a philosophical point of view anything in science that is undetectable, cannot therefore be proved or disproved, and so has no value as a proposal. If the undetectable is allowed, then one can claim, without fear of contradiction, that there are fairies at the bottom of the garden which are undetectable unless you first believe they are there,.

Another serious objection is that material bodies can only contract as a result of being physically compressed by forces. In the apparatus described, the frame separating the mirror from the observer is free and under no external forces of compression.

Another inadequacy was that it wasn't compatible with Amand Fizeau's observations regarding light refraction.

This supposed contraction in length is commonly known as the Lorentz contraction or the Lorentz-Fitzgerald contraction. A similar expression appears in Einstein's relativity but its explanation there is to do with time and has nothing to do with the motion of an ether wind. The concept of an ether had thereby become superfluous, yet Lorentz and Poincaré (the fathers of relativity) to their dying day still retained the concept of an absolute rest, which implies an ether in which one could be stationary.

So Einstein's (1905) theory established that light waves do not travel on an ether, they travel on 'nothingness'.

But this was already inherent in Maxwell's equations (1865) yet in 1890 James Clerk Maxwell whom Einstein described as the greatest scientist since Isaac Newton, asserted with great confidence that

'their can be <u>no doubt</u> that the interplanetary and interstellar spaces are not empty but are occupied by a material substance or body' *(i.e. the ether)***.**

In 1909 (4 years after Einstein's papers) Sir Oliver Lodge FRS and a very distinguished scientist, asserted in his text on

'The Ether of Space'

'The waves setting out from the sun exist in space eight minutes before striking our eyes, there <u>must necessarily</u> be in space some medium which conveys them. Waves we cannot have, unless they be waves in something.'

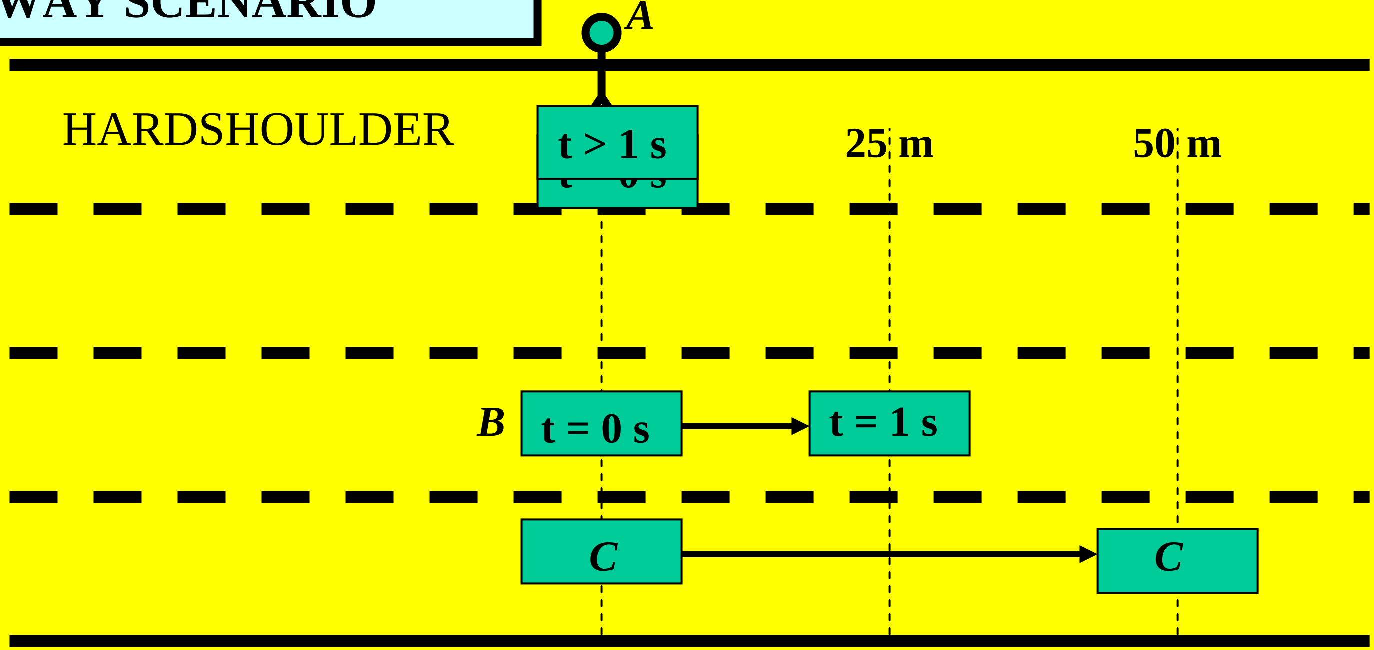
CAUTION

Claims are not true in science because someone distinguished and regarded as an authority makes them. The onus of validation rests no less on the distinguished than it does for the man in the street.

The Pope is an exception to this rule.

THE NEXT SLIDES CONSIDER A MOTORWAY SCENARIO TO ILLUSTRATE THAT IF YOU (*B*) ARE OVERTAKEN BY SOMEONE (*C*) TRAVELLING 30 MPH FASTER THAN YOU AND YOU ARE TRAVELLING 30 MPH FASTER THAN ME (*A*), IT DOES NOT MEAN THAT *C* IS OVERTAKING ME AT (30+30)=60 MPH.

A MOTORWAY SCENARIO



A **and** ***B*** **start their stopwatches (set to zero) at the positions shown.**

B **observes on his stopwatch after 1s that he is at the 25 m mark and** ***C*** **is at the 50 m mark.** ***B*** **concludes that** ***C*** **is moving at 25 m/s relative to him ($V_{CB} = 50 - 25 = 25$ m/s).**

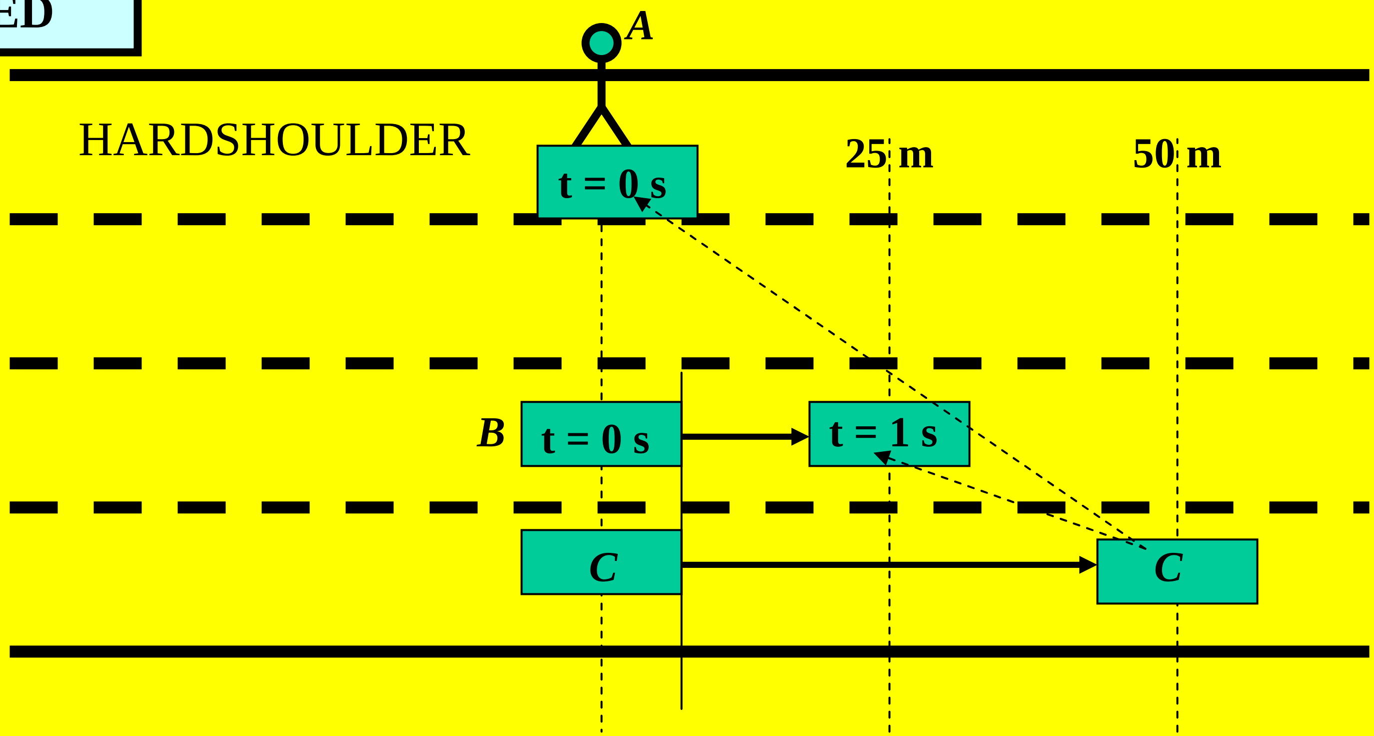
A **observes on his stopwatch after 1s that** ***B*** **is at the 25 m mark and concludes that** ***B*** **is moving at 25 m/s relative to him ($V_{BA} = 25$ m/s).**

But when ***B*** **receives the image of** ***C*** **at the 50 m mark that image has to travel a further 25 m before** ***A*** **receives it. So** ***A*** **receives this image after slightly longer than 1 s on his stopwatch so** ***A*** **concludes that** ***C*** **is moving slightly slower than 50 m/s ($V_{CA} < 50$ m/s).**

So $$V_{CA} \neq V_{CB} + V_{BA}$$

THIS INDICATES THAT WE CANNOT ADD RELATIVE SPEEDS TOGETHER BY SIMPLE ADDITION.

CONTINUED



Note that if one assumes that light travels at infinite speed then it takes no time to travel from *C* to *B* and to *A*, then when *B* observes that *C* is at the 50 m mark after 1s, so would *A*.

In that circumstance then $V_{CA} = V_{CB} + V_{BA}$

which is what we use in our everyday working because light travels so fast that any error in our calculation is negligible. A light image takes 8.3 x 10^{-8} s to travel 25 m.

When Lorentz calculated the time taken for waves to travel to and from the reflectors previously considered, he assumed that when waves travel on still water (or ether) at speed *c* and the water (ether) is traveling at speed *V*, then the waves would be seen to travel at *V*+*c*. BUT NOT SO.

As we saw from our motorway example, we cannot simply add speeds together so it is not true that

$$V_A = (V_B + V_C)$$

Observer on the bank sees waves moving with the water at this speed

Observer on the bank sees observer floating on and moving with the water at this speed (V_B)

Floating observer moving on the water sees waves moving at this speed (V_C)

As we shall see, due to the Doppler effect and time dilatation this requires the following correction

$$V_A = (V_B + V_C) \times \left[\frac{1}{1 + V_B V_C / c^2} \right]$$

Had Lorentz known this then when adding the speed of the waves $V_C = c$ to the speed of the stream $V_B = V$ he would have estimated that the speed of the waves V_A relative to A (the earth observer standing on the bank) would be

$$V_A = (V + c) \times \left[\frac{1}{1 + Vc/c^2} \right] = c\frac{V + c}{V + c} = c$$

If they had been light waves he would have estimated them to move at exactly the same speed as in the still water (still ether) in spite of the fact that they are being carried along by the water (ether). All observers observe light waves to move at the same speed. But then had Lorentz realised this in 1902 & 1904 he would have been several steps ahead of Einstein (1905).

Lorentz also assumed that the time taken for the waves to travel from the observer *B* moving with the stream, to the transverse reflector and then back to the earth observer *A*, would be the same as that for a stationary earth observer *A* (i.e $t_A = \tau_B$, time is the same for everyone). What he did not know was that this is not so and as we shall see, that in fact as we have already seen

$t_A = \frac{\tau_B}{\gamma}$ **so when he calculated that** $\tau_B = \frac{2L\gamma}{c}$ **then for *A*** $t_A = \frac{\tau_B}{\gamma} = \frac{2L\gamma}{\gamma c} = \frac{2L}{c}$

Remember he had calculated that the time for waves to travel to the longitudinal reflector and back was $\frac{2L\gamma^2}{c}$ **because they traveled at V+c in one direction and V-c in the opposite direction. But as we have just seen, if they are light waves they would be observed to travel at speed c in both directions so the time should be simply** $\frac{2L}{c}$

If we now take the difference between the times *A* should observe for waves to travel to the transverse reflector and back from that to the longitudinal reflector and back, we obtain

$$\frac{2L}{c} - \frac{2L}{c} = 0$$

which is why no difference could be observed and it has nothing to do with Lorentz's length contraction idea or the ether wind and only to do with the fact that observed time duration is not the same for observers in relative motion (e.g *A* and *B*) as we shall see.

Note however that as in our water analogy, Lorentz assumed that if waves travel on the water at speed *c* and the water is moving at speed *V* relative to the bystander *A* on the bank, then the waves would be moving away at speed (*V*+*c*) relative to the bystander (which as we saw from our motorway analogy would only be true if images traveled at infinite speed). This was in spite of the fact that Römer's observations (1676) on Jupiter's moon Io 226 years earlier, estimated that light travels at a finite speed in excess of 225,000 km/s.

These false assumptions therefore completely invalidated the hypotheses of Lorentz and Fitzgerald that the lengths of objects contract in the direction of a moving ether stream. Nevertheless under the patronage of Henri Poincaré (the father of relativity), Hendrik Lorentz was awarded the Nobel prize for his contribution to work in this area in (1902).

Nevertheless such an apparent contraction figures in Einstein's relativity (but for an entirely different reason concerned with time and doesn't imply any physical contraction).Yet this feature still bears the imprimatur of Lorentz and Fitzgerald.

No one seems to have considered the implications of Römer's (1676) observation on the finite speed of light as in our motorway analogy. Perhaps it's because there were no motorways in those days ?

You have probably heard that no object can travel faster than the speed of light (1975 standard)

***c* = 299,792,458 *m/s* (say 300,000 *km/s*) .**

Does this mean that we can never observe an object to move more that 300,000 *km* in 1 *s* (or 0.3 *km* in 1 *μs* which is the same speed) ?

NO IT DOES NOT MEAN THIS !

It means that the relativistic speed (*V*) of an object is always less than the above speed of light

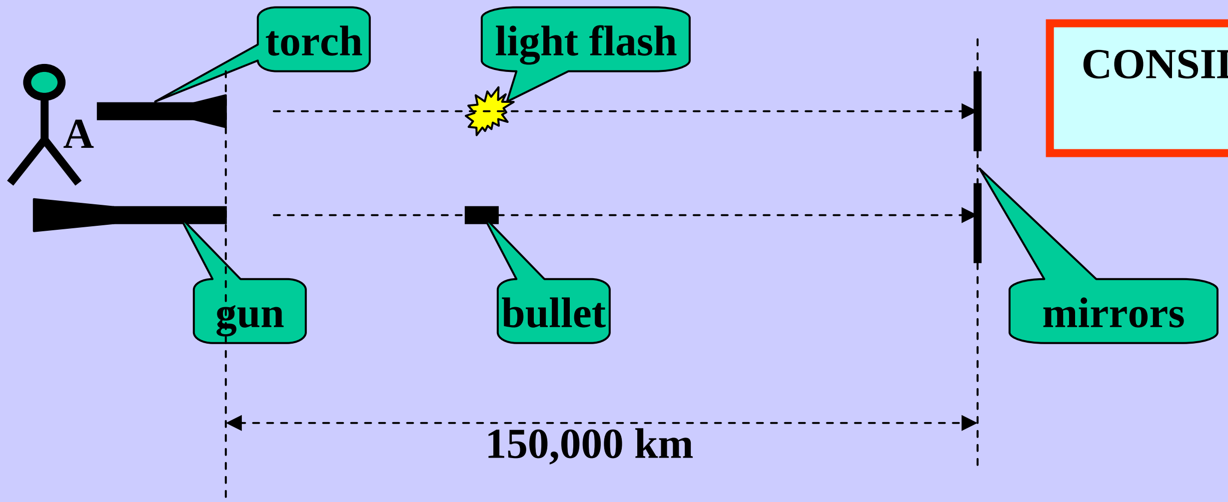
$$V \equiv \frac{x}{t - r/c} \leq 300{,}000 \;\; km/s$$

But we can and do observe objects to move so that their apparent speed

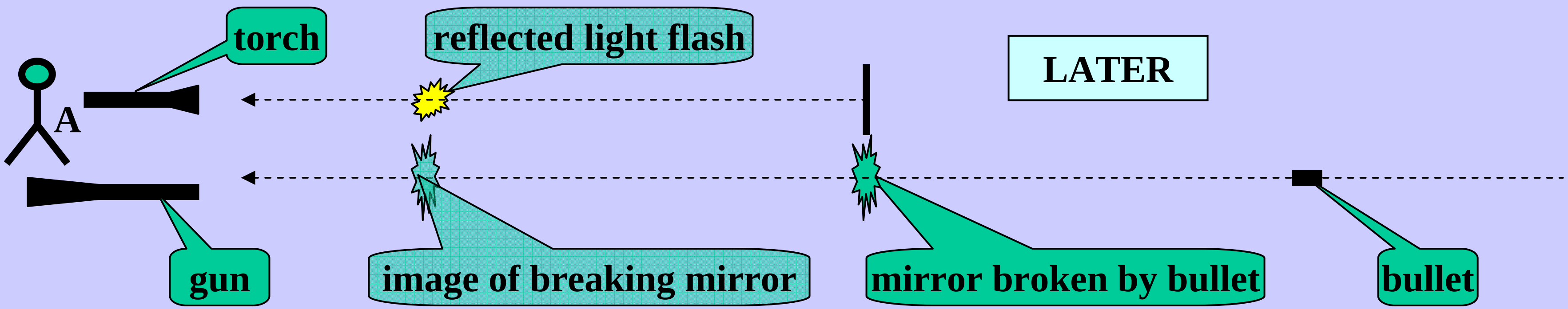
$$\frac{x}{t} > 300{,}000 \; km/s$$

if it's in approach to us. As you will be seen on a subsequent slide, something approaching at the speed of light appears to be moving at infinite speed.

CONSIDER THE FOLLOWING SCENARIO



Suppose *A* has a gun that can fire bullets at the same speed as (the flash of) light from the torch.



The light flash travels to the mirror and reflects back to *A* in 1 *s* on *A*'s stopwatch, a total distance of 300,000 *km*. He concludes that light travels at 300,000 *km/s*.
The image of the breaking mirror also travels with the flash back to *A* in 1 s. He observes that the bullet traveled 150,000 *km* in 1*s*. He concludes that the bullet traveled at 150,000 *km/s*.

But the light flash and the bullet both traveled together to the mirrors at the same speed ??

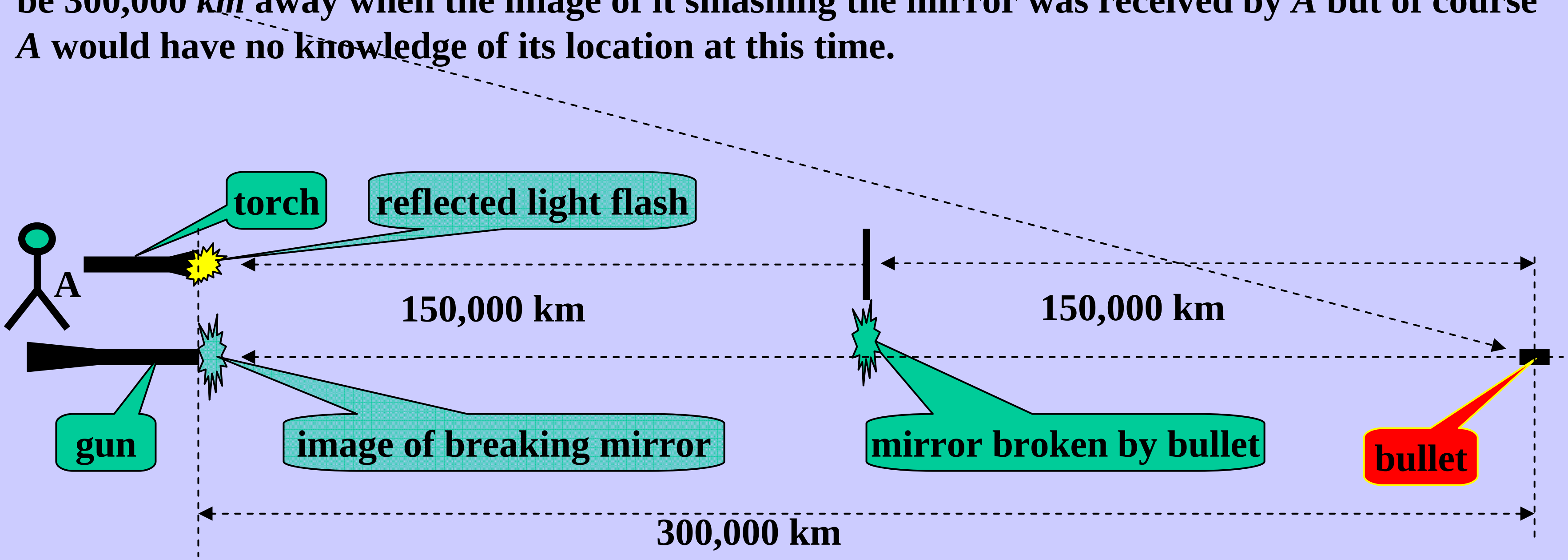
CONTINUED

So when an object travels away from you at the speed of light it is observed to travel at 150,000 *km* in 1 s (i.e. 150,000 *km/s*)

But its relativistic speed is

$$V \equiv \frac{x}{t - r/c} = \frac{150{,}000}{1 - 150{,}000/300{,}000} = \frac{150{,}000}{1 - 1/2} = 300{,}000 \;\; km/s$$

But assuming that the bullet wasn't slowed down by the impact with the mirror then it would be 300,000 *km* away when the image of it smashing the mirror was received by *A* but of course *A* would have no knowledge of its location at this time.



CAUTION

The word speed in relativity usually refers to relativistic speed

$$V = \frac{x}{t - r/c}$$

NOT what you would normally think speed to be (observed distance moved divided by observed time taken)

$$V = \frac{x}{t}$$

which (if referred to at all) is referred to as apparent speed

WE NEXT CONSIDER HOW THE TIME INTERVAL BETWEEN TWO EVENTS MAY BE DIFFERENT FOR PEOPLE WHO ARE MOVING RELATIVE TO US.

We shall be using the words 'sender' (or 'emitter') of signals and 'receiver' of those signals.

The words 'sender' or 'emitter' do not imply that some transmitting device is necessarily being used to send signals. We see objects usually because they are reflecting light to us so they are sending us light signals (images) although they are not actively emitting light they are still the sender of the image. We are the receiver of these signals (images) simply by viewing them. So these terms need not imply any physical transmitting and receiving equipment.

The word signal here just indicates that some information has been transmitted by electromagnetic waves (e.g. images, radio etc).

NOW FOLLOW SOME APPLIED EXAMPLES

Note that in what follows, the term directly means that *B* is receding from *A* in line of sight.

In these cases V and V_r are the same thing so $V = V_r$

Suppose you are receding directly from me at 80% of the speed of light (i.e. $V/c = V_r/c = 0.80$) and you send me the message that you have just started your stopwatch, then send me a second message 1 second later on your stopwatch saying that your stopwatch now reads 1 second. I start my stopwatch on receiving your first message, then on receiving your second message, I observe that my identical stopwatch reads

$$t_A = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}} \times t_B \quad = \frac{1+0.8}{\sqrt{1-(0.8)^2}} \times 1 = 3 \text{ seconds}$$

Any textbook will tell you that the answer is $= \dfrac{1}{\sqrt{1-(V/c)^2}} = 1.667 \text{ seconds}$ **???**

This is because they are not referring to what you will actually observe on your stopwatch but to the relativistic time τ_A. If you remember

$\tau \equiv t - r/c$ **and** $V_r \equiv r/\tau$ **from which (with a little tedious algebra) we obtain**

$$\tau = \frac{t}{1+V_r/c} = \frac{3}{1+0.8} = 1.667$$

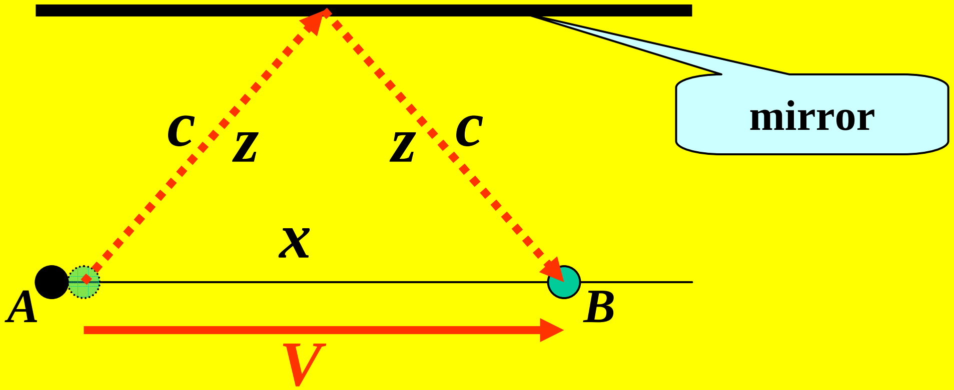
in agreement with the textbook (but the book is unlikely to tell you that this isn't what you will actually observe).

Suppose you are passing me and now receding directly from me at the speed of light (i.e. $V/c = +1$). We both start our stopwatches as you pass. As before you send me a message that your stopwatch now reads 1 second. On receiving your message I observe that my identical stopwatch reads

$$t_A = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}} \times t_B = \frac{1+1}{\sqrt{1-1^2}} \times 1 = \frac{2}{0} = \infty$$

I will wait forever and never receive your second message. How is this ????

If you remember the ratio of *B*'s speed (*V*) to the speed of the signal (*c*) is the distance the observer *A* sees the light signal move ($2z$) to the mirror and back, in the same time as he sees you move the distance *x*. If you are moving at the speed of light then $x = 2z$.



But for *x* to equal $2z$, the base of the triangle would have to be infinitely long. You would have to travel an infinite distance to make $x = 2z$. So the light traveling at 300,000 km/s would take an infinite time to get back to me.

Suppose you are approaching me directly at 80% of the speed of light (i.e. $V_r/c = V/c = -0.80$) and as before you send me the message that you have just started your stopwatch, then send me a second message 1 second later on your stopwatch saying that your stopwatch now reads 1 second. I start my stopwatch on receiving your first message, then on receiving your second message, I observe that my identical stopwatch reads

$$t_A = \frac{1+(V_r/c)}{\sqrt{1-(V/c)^2}} \times t_B \quad = \frac{1+(-0.8)}{\sqrt{1-(-0.8)^2}} \times 1 = \frac{1}{3} \text{ seconds}$$

But any textbook will tell you that the answer is $= \dfrac{1}{\sqrt{1-(V/c)^2}} = 1.667 \text{ seconds}$ **???**

This is the same value as when you were receding from me. Because again they are not referring to what you will actually observe on your stopwatch but to the relativistic time τ_A.

What we are saying here is that for example in the first case, when the light signal travels to *B*'s mirror and back, *A* will observe these same two events to have traveled to *A*'s similarly placed mirror and back three times. We are using the mirror and light signal as a clock device which is independent of any physical moving parts.

LATER WE SHALL CONSIDER THE CASE WHEN YOU ARE APPROACHING ME AT THE SPEED OF LIGHT

You have no doubt heard in relation to special relativity that moving clocks run slow.

So, is it true ?

A fundamental premise of the theory (as stated earlier) is that the meaning of relativity implies that any two observers moving at constant velocity one relative to the other will not observe any difference to results of any identical experiment carried out by each of them.

We are then stating that two observers in constant relative motion observing their identical clocks will find no difference in their behaviour therefore it would be a contradiction to say that one clock runs slower than the other. So *a priori* the answer to the question 'do moving clocks run slow' must be

NO.

Why then are we told that they do ?

Our results so far have shown that if you are receding directly from me, then if we each observe two events occurring at your location, your clock will tell a <u>lesser</u> time interval between the two events than my clock.

This statement says exactly what it means. It does not say that your clock is running slow !

But one might say that the situation is AS IF your clock is running slow.

THIS IS ALL VERY WELL BUT

If we compare this with the case when you are in direct approach to me, our results have shown that the time observed on your clock between two events occurring at your location will be greater than I observe it to be on my clock.

So in this case it would not even be correct to say that it is AS IF your moving clock was running slow, rather that it was AS IF your clock is running fast.

So our results are saying definitively that the answer to the question 'do moving clocks run slow ?' is NO and the answer to the question 'is it AS IF moving clocks run slow ?' is ONLY IF THEY ARE RECEDING.

As you have by now probably guessed the explanation is that what they are saying is that in terms of the relativistic time (τ) it is AS IF your moving 'CLOCK' runs slow because (as we have seen) your value of τ will be less than mine, whether you are in approach or in recession from me, and whether direct or not.

But this overlooks the fact that τ CANNOT BE OBSERVED ON ANY CLOCK DEVICE.

Remember that τ is the time we estimate that the information left the sender, $\tau = t - r/c$ and requires the actual observations of received clock time t, distance x and the knowledge of the information transmission speed c.

ONLY IF YOU WANT TO KNOW

THE TWIN PARADOX

This poses the scenario concerning two twins *A* and *B*. *B* goes on a journey and subsequently returns to *A*. They then compare the duration of the journey on each others stopwatch.

We can use our previously deduced time relationship to calculate that from *A*'s point of view *B*'s stopwatch will show a lesser duration than *A*'s on *B*'s return. So on return *B* is younger than his twin *A*. But from *B*'s point of view it was *A* who undertook the journey so the calculation will show for him similarly that his twin *A* is younger than him.

Well they can't each be younger than the other. That's the paradox.

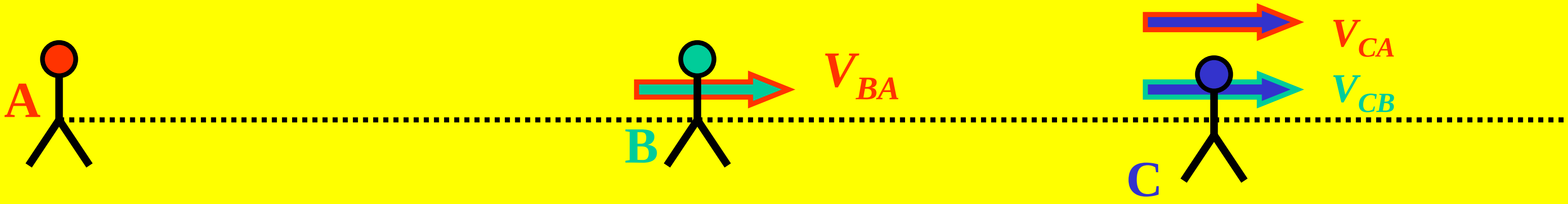
But the time relationship we use to make this calculation is based on special relativity A FUNDAMENTAL PREMISE OF WHICH WAS THAT *A* AND *B* HAVE A CONSTANT VELOCITY RELATIVE TO EACH OTHER (i.e. NO ACCELERATION). This was because the physical behaviour of anything (including clocks) taking place at *A* and *B* when compared, are always found to be the same ONLY if they are in constant velocity one relative to the other, otherwise it has nothing to say.

For *A* and *B* to separate requires that there be relative acceleration between them and further accelerations are required if they are to return to each other. So the very conditions on which the time relation is based are broken and special relativity has nothing to say about any result you may calculate using it in these circumstances. The time relationship has been used inappropriately BUT a similar argument is used to deduce the relativistic laws of motion starting from Newton's low speed law !!!

If *B* ejects matter to accelerate away from *A* then his accelerometer will register this so while their relative acceleration remains mutual, *A*'s accelerometer registers no change. There is a difference between them. Experiments carried out by *A* will no longer be confirmed by *B*.

STUDY No 1 : SPEED

Consider Observers A, B and C



B is moving away from A at a speed V_{BA}

C is moving away from B at a speed V_{CB}

C is moving away from A at a speed V_{CA}

C sends out two signals t_C seconds apart on his stopwatch.

We know from our previous work that:

B will receive the signals at a time interval of

$$t_B = \frac{1+(V_{CB}/c)}{\sqrt{1-(V_{CB}/c)^2}} \times t_C$$

A will receive the signals at a time interval of

$$t_A = \frac{1+(V_{CA}/c)}{\sqrt{1-(V_{CA}/c)^2}} \times t_C$$

B sends on the signals he receives to A so

equal **substitute**

A will receive these signals at a time interval of

$$t_A = \frac{1+(V_{BA}/c)}{\sqrt{1-(V_{BA}/c)^2}} \times t_B$$

CONTD

Writing in the equality we obtain and making the substitution for t_B we obtain

$$t_A = \frac{1+(V_{CA}/c)}{\sqrt{1-(V_{CA}/c)^2}} \times t_C = \frac{1+(V_{BA}/c)}{\sqrt{1-(V_{BA}/c)^2}} \times t_B = \frac{1+(V_{BA}/c)}{\sqrt{1-(V_{BA}/c)^2}} \times \frac{1+(V_{CB}/c)}{\sqrt{1-(V_{CB}/c)^2}} \times t_C$$

So the the two red boxes are equal in value.

$$\frac{1+(V_{CA}/c)}{\sqrt{1-(V_{CA}/c)^2}} = \frac{1+(V_{BA}/c)}{\sqrt{1-(V_{BA}/c)^2}} \times \frac{1+(V_{CB}/c)}{\sqrt{1-(V_{CB}/c)^2}}$$

We can re-arrange this algebraically to extract the value of V_{CA}

$$V_{CA} = (V_{CB} + V_{BA}) \times \left(\frac{1}{1+V_{CB}/c \times V_{BA}/c} \right)$$

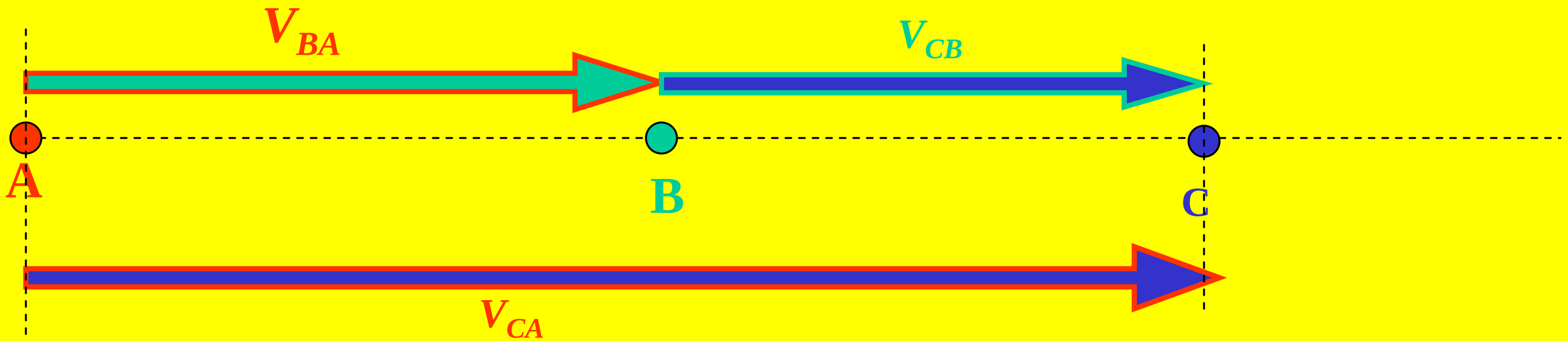
This is the relationship we were looking for earlier in order to be able to add the velocity of the stream to the velocity of the waves, taking account of the time difference between them.

We can read this as

$$V_{CA} = \left(V_{BA} + V_{CB}\right) \times \left(\frac{1}{1 + V_{CB}/c \times V_{BA}/c}\right)$$

The familiar sum of the relative velocities (due to Galileo)

The relativistic correction to the Galilean result

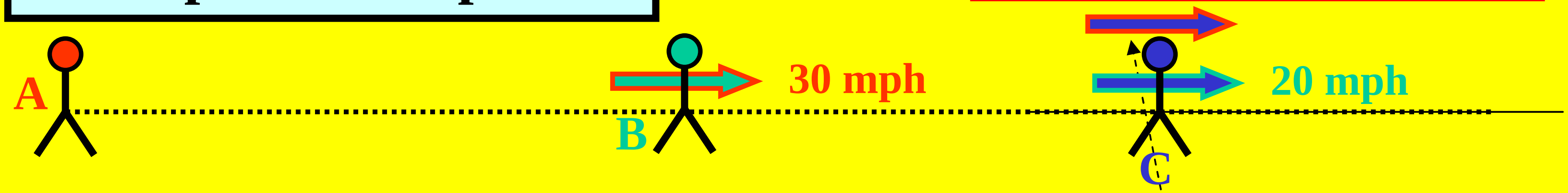


This fundamental equation shows how the relativistic speed of C to A is related to the speed of C to B and of B to A. We cannot use the simple Galilean addition of speeds as Lorentz did.

WE NEXT APPLY THIS TO A FEW EXAMPLES

Low Speed Example

49.999999999999999967 mph



B is moving away from A at a speed 30 *mph* = 0.00833 *mps*

C is moving away from B at a speed 20 *mph* = 0.00555 *mps*

According to Galileo C is moving away from A at (30+20) = 50 *mph*

Speed of light '*c*' =186,000 *mps*

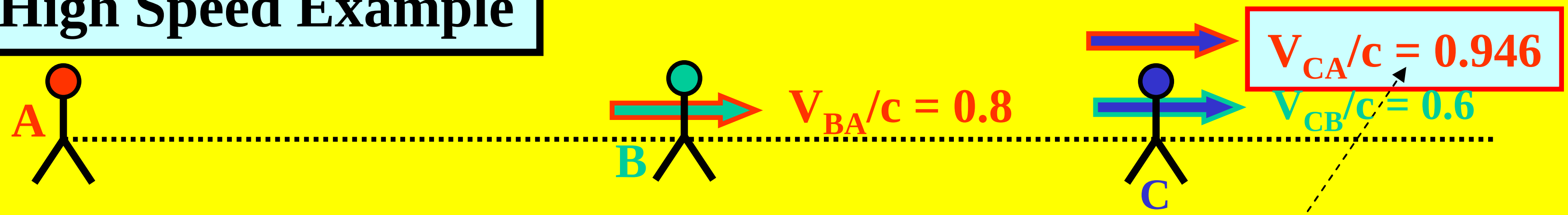
According to our previous reasoning this requires a relativistic correction

$$\times\left(\frac{1}{1+0.00833/186000\times0.00555/186000}\right)=1-0.00000000000000134$$

So the answer should be

$$50\times\left(1-0.00000000000000000013\right)=49.999999999999999967\ \textbf{\textit{mph}}$$

High Speed Example



B is moving away from A at 0.8 the speed of light

C is moving away from B at 0.6 the speed of light

According to Galileo C is moving away from A at = 0.8+0.6 = 1.4 times the speed of light.

But according to our previous reasoning this requires a relativistic correction

$$\times\left(\frac{1}{1+0.8\times 0.6}\right)=0.676$$

So the answer is in fact

$$1.4\times 0.676=0.946$$ **times the speed of light**

Whatever the speeds of B to A or of C to B, the speed of C to A will always be less than the speed of light . But remember these are the relativistic speeds NOT the observed speeds. Observed (apparent) speeds of objects in approach can exceed the speed of light as illustrated on a subsequent slide.

Dividing our velocity relationship below by *c* we can re-write this entirely in terms of fractions

$$V_{CA} = \left(V_{CB} + V_{BA}\right) \times \left(\frac{1}{1 + V_{CB}/c \times V_{BA}/c}\right)$$

thus $$\frac{V_{CA}}{c} = \left(V_{CB}/c + V_{BA}/c\right) \times \left(\frac{1}{1 + V_{CB}/c \times V_{BA}/c}\right)$$

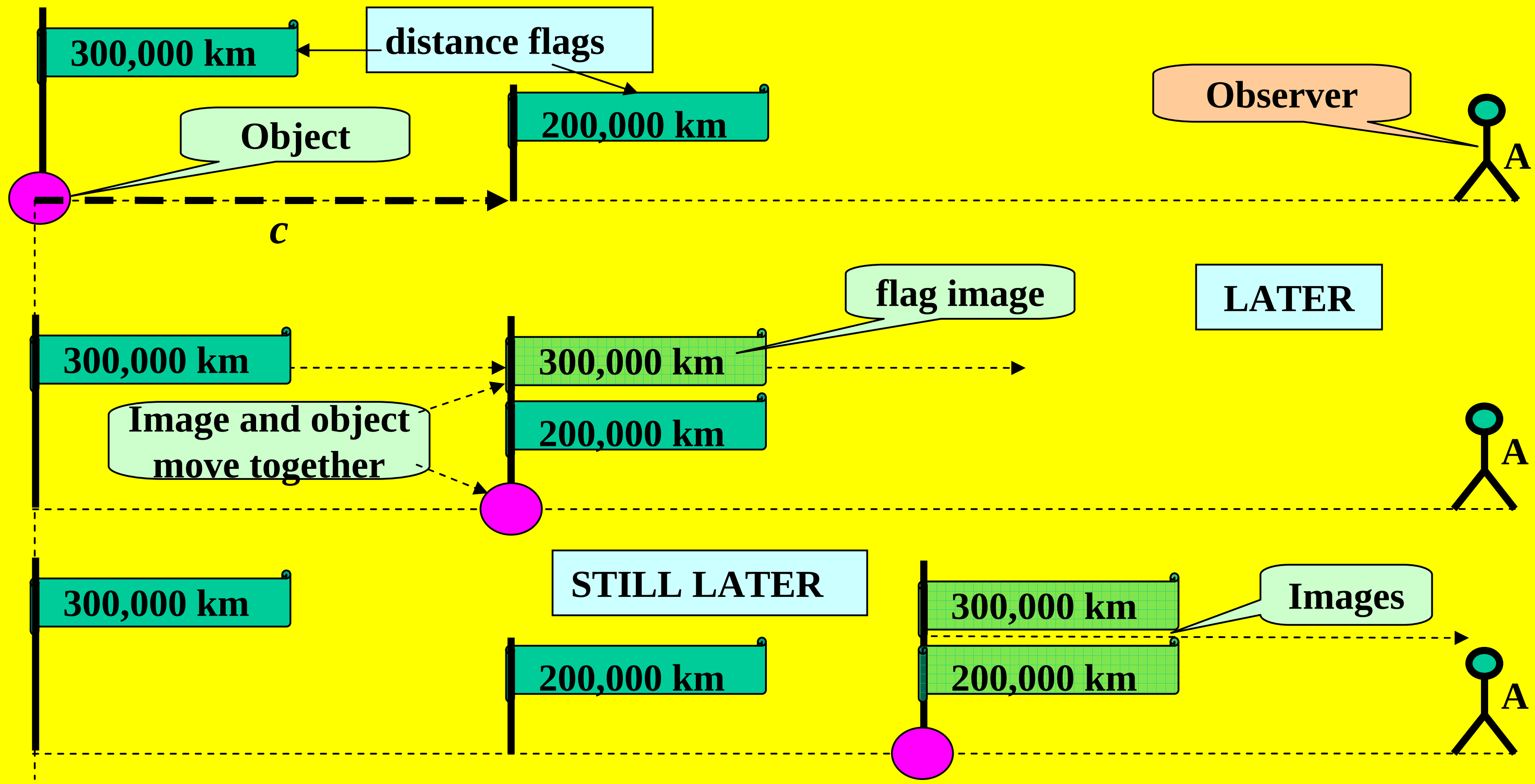
The reason that V_{CA}/c will always be less than unity (i.e. the speed of light) is because V_{CB}/c, V_{BA}/c are less than the speed of light (they are fractional values) and any fractional values substituted in the expression on the left above always results in another fraction (i.e.less than the speed of light)

e.g. $$(0.9 + 0.9) \times \frac{1}{1 + 0.9 \times 0.9} = \frac{1.8}{1 + 0.81} < 1$$

Note that if *C* is passing *B* at the speed of light $V_{CB}/c = 1$ and *B* is receding from *A* at 99% of the speed of light V_{BA} =0.99 then

$$V_{CA} = (1 + 0.99) \times \frac{1}{1 + 1 \times 0.99} = 1$$

and the speed of *C* to *A* is still only the speed of light (i.e. $V_{CA}/c = 1$)



Suppose that the object moves at the speed of light '*c* = 300,000 km/s, the same speed as light images.

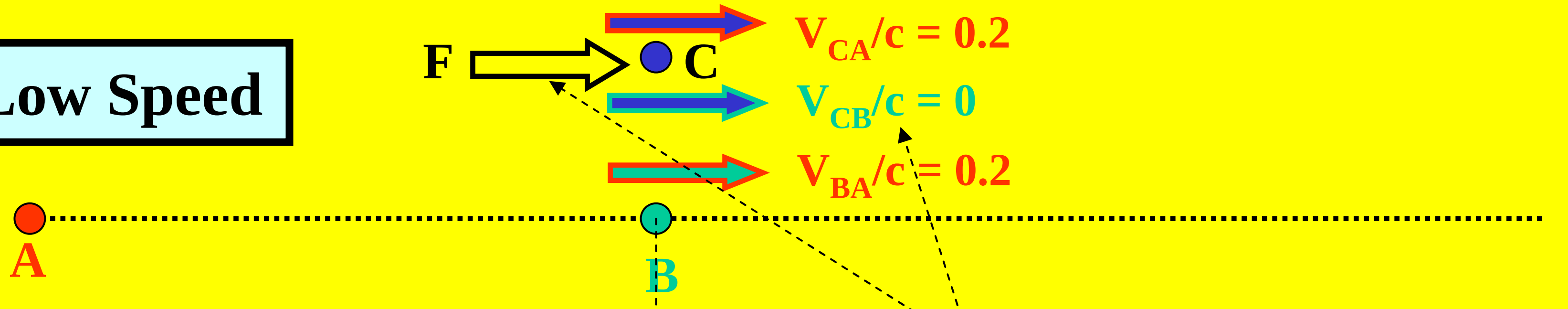
***A* starts his stopwatch when he receives the image of the object at the 300,000 *km* flag and stops it when he receives the image of the object at the 200,000 *km* flag. But the two images arrive together so *A* observes the time interval to move the 100,000 *km* from 300,000 *km* flag to the 200,000 *km* flag to be zero. So the apparent speed of the object is**

$$\frac{x}{t} = \frac{100,00}{0} = \boxed{\infty}$$ **but the relativistic speed is** $$V = \frac{x}{t - r/c} = \frac{100,00}{0 - \left(-100,00\right)/c} = c = 300,000 km/s$$

STUDY No 2 :MASS

We first deal with the ‘why’s and then the ‘wherefore’s.

Low Speed

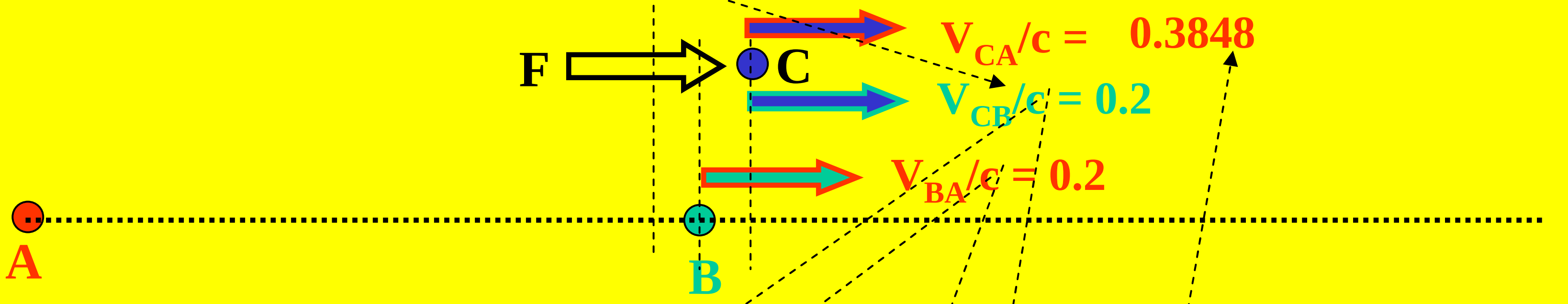


B is moving at constant speed of 0.2 the speed of light away from A

C is increasing in speed because it has a constant force F applied to it

but at this moment is moving at the same speed as **B so** $V_{CB}/c = 0$ **and** $V_{CA}/c = 0.2$

One second later on B's clock the force F has increased the speed of C, and C is now moving at speed of 0.2 the speed of light relative to B



The speed of C relative to A is now **with relativistic correction**

$$V_{CA}/c = (0.2 + 0.2) \times \frac{1}{1 + 0.2 \times 0.2} = 0.3848$$

ONLY IF YOU WANT TO KNOW

HOW NEWTON'S LAWS OF MOTION APPLY TO *A*

The next two slides are mathematical and develop how the form of Newton's laws of motion applying to *B* viz.

$$F = m_C \times \frac{dV_{CB}}{d\tau_B} \qquad \textbf{and} \qquad F = m_C \times \frac{d^2\ell_B}{d\tau_B^2}$$

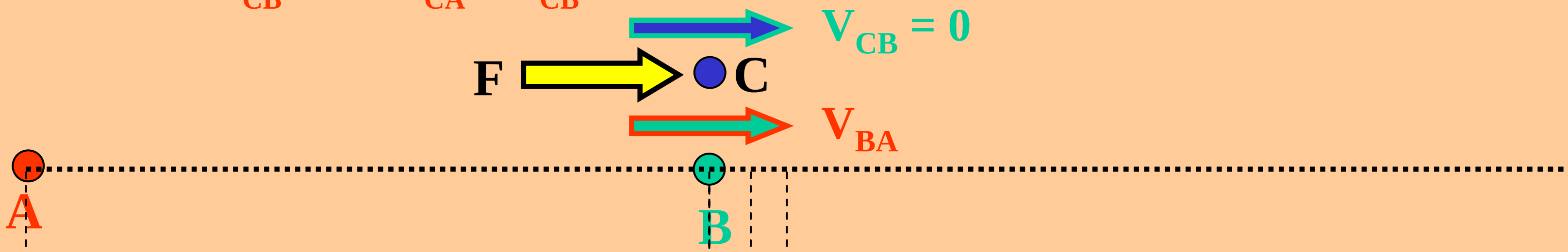
will apply to *A*. It considers a mass m_C being accelerated by a force *F* which at the moment is moving at the same speed as an observer *B* who moves at constant speed relative to *A*. It considers events only for the next indefinitely short period of time during which the mass m_C will acquire only a negligible speed relative to *B* so during this interval Newton's laws of motion will apply for *B* and because *B* is moving at constant velocyt relative to *A* the time relationships we have developed will apply between *B* and *A*. Our task is to show that when the force is in the direction of motion

$$\frac{dV_{CB}}{d\tau_B} = \gamma^3 \frac{dV_{CA}}{d\tau_A} \qquad \textbf{and when transverse that} \qquad \frac{d^2\ell_B}{d\tau_B^2} = \gamma^2 \frac{d^2\ell_A}{d\tau_A^2}$$

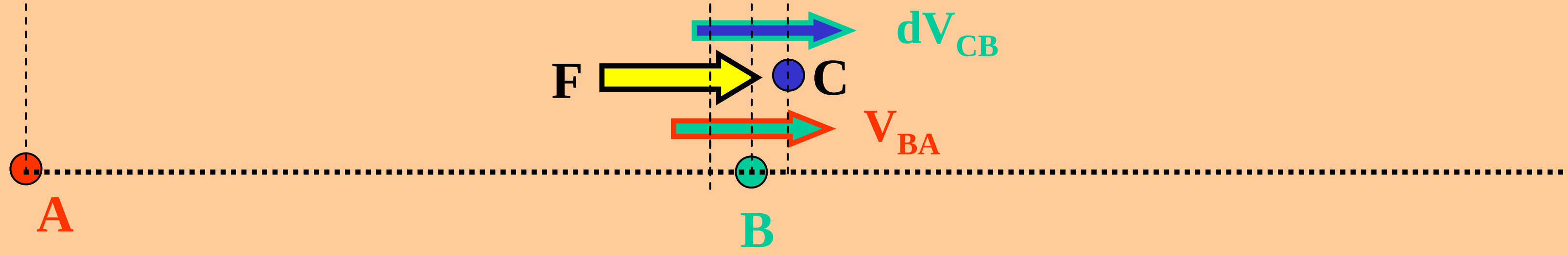
in agreement with Einstein's 1905 paper.

Observer *B* moves at constant speed relative to *A* so V_{BA} = constant

C is accelerating because it has a force applied to it but at <u>this moment</u> is moving at the same speed as *B* i.e $V_{CB} = 0$ so $V_{CA} = V_{CB}$



A moment $d\tau_B$ later C is now moving faster than B by a very slight amount dV_{CB}



Because the speed of *C* relative to *B* is so small, Newton's law of motion will be valid during the short period $d\tau_B$ (we are considering the values of $d\tau$ and dV to be indefinitely small) so

$$F = m_C \times a_{CB}$$

where a_{CB} is the acceleration of C relative to B i.e. $a_{CB} = \dfrac{dV_{CB}}{d\tau_B}$

But what we are seeking is a relationship between the force F and the motion of *C* relative to *A*. We already have a relationship between $d\tau_A$ and $d\tau_B$ so we now need a relationship between dV_{CB} and dV_{CA}.

WHEN THE FORCE IS APPLIED IN THE DIRECTION OF MOTION

We have previously developed the speed relationship between A, B and C, it is

$$V_{CA} = \frac{V_{CB} + V_{BA}}{1 + V_{CB}/c \times V_{BA}/c}$$

So applying this to our current scenario. In the first instant $V_{BA} = V_{CA}$ because A and B are moving at the same speed. A moment $d\tau_B$ later *C* is moving relative to *B* at a very small speed dV_{CB} so Newton's law can be applied between A and B during this period

$$V_{CA} + dV_{CA} = \frac{dV_{CB} + V_{CA}}{1 + dV_{CB}/c \times V_{CA}/c}$$

which for dV_{CB} very small approximates to (and which becomes exact as $dV_{CB} \to 0$)

$$V_{CA} + dV_{CA} = \left(dV_{CB} + V_{CA}\right)\left(1 - dV_{CB}/c \times V_{CA}/c\right)$$

$$= V_{CA} + \left[1 - \left(\frac{V_{CA}}{c}\right)^2\right] dV_{CB} \quad + \textbf{(a negligible term in } dV_{CB}^2\textbf{)}$$

hence

$$dV_{CA} = \left[1 - \left(\frac{V_{CA}}{c}\right)^2\right] dV_{CB} = \frac{dV_{CB}}{\gamma^2}$$

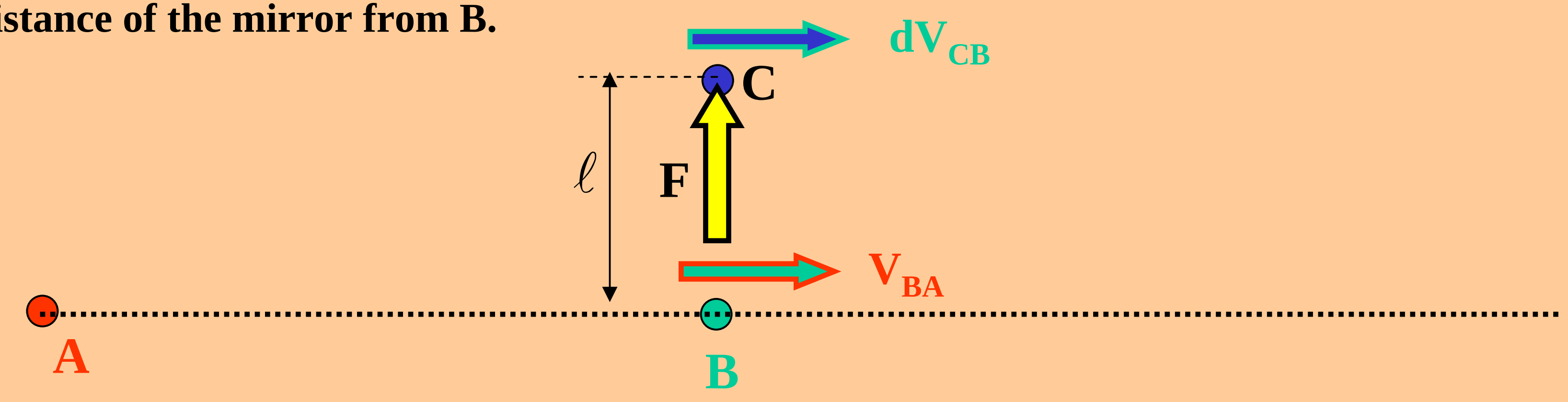
is the relationship we were looking for. We know that $d\tau_A = \gamma\, d\tau_B$ **so** $\dfrac{dV_{CB}}{d\tau_B} = \gamma^3 \dfrac{dV_{CA}}{d\tau_A}$

So for A the equivalent of Newton's law is

$$\boxed{F} = m_C \frac{dV_{CB}}{d\tau_B} = \boxed{\gamma^3 m_C \frac{dV_{CA}}{d\tau_A}}$$

WHEN THE FORCE IS APPLIED TRANSVERSE TO THE DIRECTION OF MOTION

We designate distances transverse to the direction of motion by the symbol ℓ as we did for the distance of the mirror from B.



As before Newton's law $F = m_C \times \dfrac{d^2\ell}{d\tau_B^2}$ **applies to *B* for the reason's previously given.**

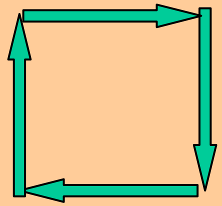
We know that $d\tau_A = \gamma\, d\tau_B$ **so** $d\tau_A^2 = \gamma^2\, d\tau_B^2$ **therefore** $F = m_C \times \gamma^2 \dfrac{d^2\ell}{d\tau_A^2}$

and the speed in the direction of ℓ **relative to *A* is** $V_\ell \equiv \dfrac{d\ell}{d\tau_A}$ **therefore**

$$F = m_C \times \gamma^2 \frac{dV_\ell}{d\tau_A}$$

ONLY IF YOU WANT TO KNOW

A major problem with the previous two results is that they were both arrived at by the same logical reasoning so if one is true the other is true. But they do not satisfy the conservation of momentum which is regarded as an over-riding law of nature (as is the conservation of energy). Conservation of momentum says that if we apply unit force for one unit of time sequentially to a given mass as below



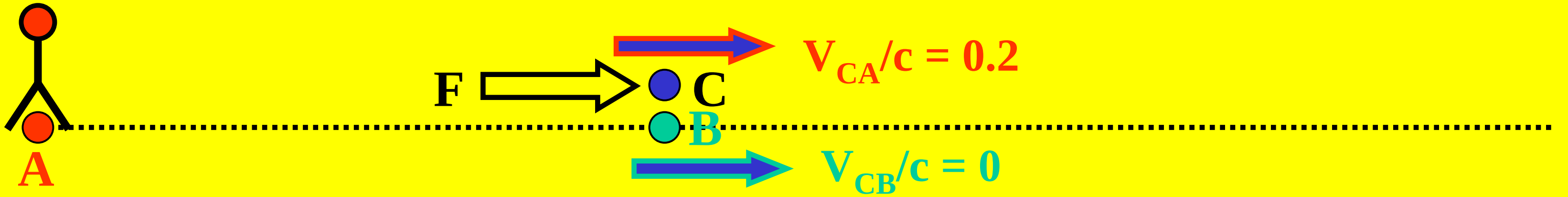
then the mass should have the same velocity after the sequence as it had before it.

But the equations we have developed (as Einstein did) do not produce this result. If the equation we derived when the force is applied in the direction of motion is true then the equation for the force when applied transverse to the direction of motion that will conserve momentum is not the one we derived

$$F = m_C \times \gamma^2 \frac{dV_\ell}{d\tau_A} \qquad \textbf{BUT} \qquad F = m_C \times \gamma \frac{dV_\ell}{d\tau_A}$$

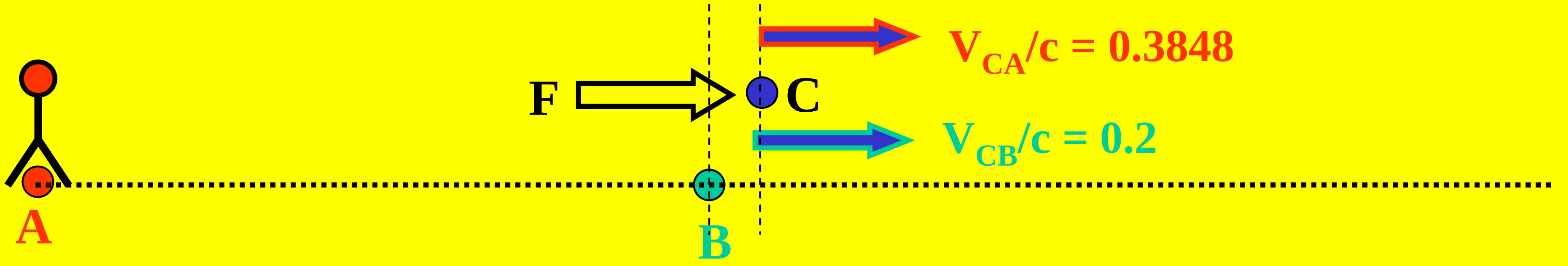
There is no comment in Einstein's paper regarding this point. A.P.French in his book 'Special Relativity' raises the issue without satisfactory resolution. There appears to be an internal inconsistency in the argument. We have no more reason to accept the first equation as true than to accept the second - another PADADOX.

Low Speed Recap



B moves at constant speed of 0.2. At this moment B and C are moving at the same speed.

Because C has a force applied to it it accelerates away from B.

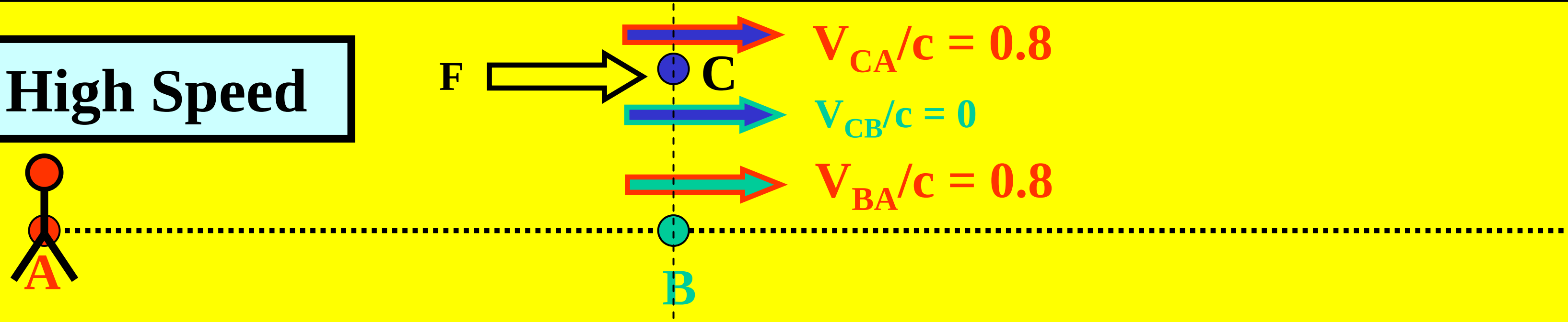


So now the speed of C relative B has increased from 0 to 0.2 an increase of 0.2

But as we have just seen the speed of C relative to A has increased from 0.2 to 0.3848 an increase of 0.1848.

WE NOW CONSIDER A SIMILAR SEQUENCE OF EVENTS WHEN *B* IS MOVING AT A CONSTANT HIGH SPEED OF 0.8 (in the last slide it was 0.2). WE APPLY THE SAME FORCE AS BEFORE SO THAT *C*'s SPEED IS MOVING FASTER THAN *B* BY 0.2 AGAIN.

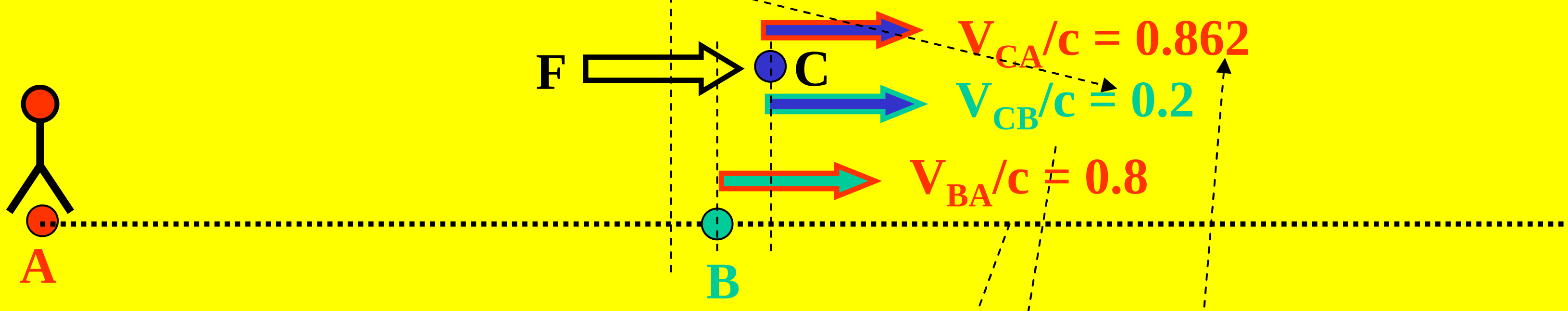
High Speed



B is moving at constant speed of 0.8 of the speed of light away from A

C is increasing in speed because it has a constant force F applied to it *but at this moment is moving at the same speed as* B so $V_{CB}/c = 0$ and $V_{CA}/c = 0.8$

One second later on B's clock, Newton tells us that the same force F will increase the speed of C by the same amount as before, so C is now moving at speed of 0.2 of the speed of light relative to B

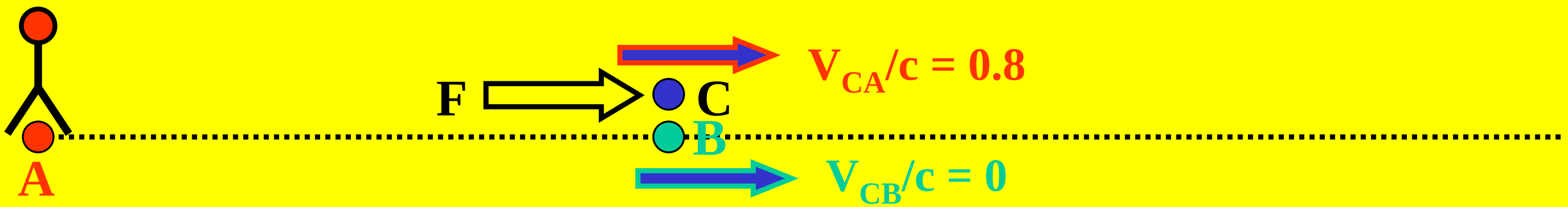


The speed of C relative to A is now

with relativistic correction

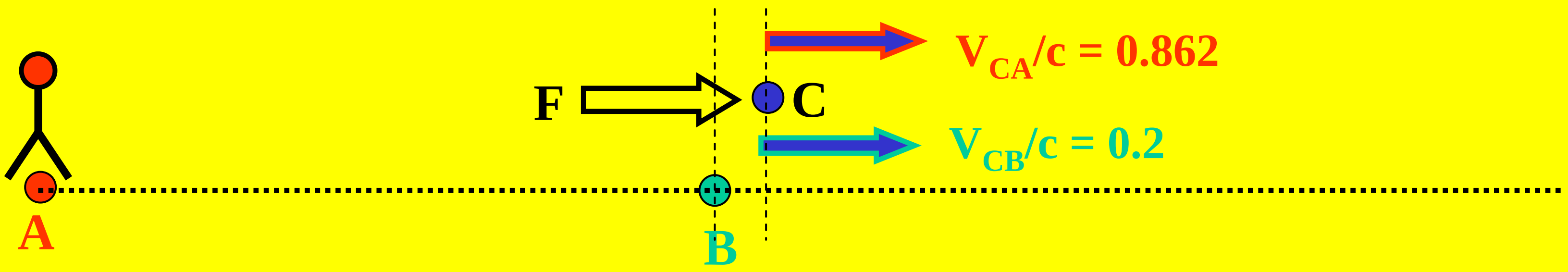
$$V_{CA}/c = (0.8 + 0.2) \times \frac{1}{1 + 0.8 \times 0.2} = 0.862$$

High Speed Recap



B moves at constant speed of 0.8. At this moment B and C are moving at the same speed.

Because C has a force applied to it it accelerates away from B.



So now the speed of C relative B has increased from 0 to 0.2 an increase of 0.2

But as we have just seen the speed of C relative to A has increased from 0.8 to 0.862 an increase of only 0.062.

From the previous two examples, the first at low speed and the second at high speed, we observe that when a force *F* is applied for *B*'s time t_B=1 s to the mass C, then when C is already moving at Low Speed (V_{CA}/c = 0.2)
***A* observes the mass to increase speed by (0.3848 - 0.2) = 0.1848**

High Speed ($V_{C\,A}$/c = 0.8)
***A* observes the mass to increase speed by only (0.862 - 0.8) = 0.062**

When the object moves at high speed *A* observes that the action of the force is to increase the objects speed far less than when it moves at low speed. In other words, we could say, it exhibits much higher inertia when moving at high speed.

Unfortunately this inertia is often referred to as relativistic mass but often with the word relativistic omitted so you are left thinking the mass (i.e. the quantity of matter) of the object increases at high speed as if it was creating more nucleons.

Newton's law of motion tells us that in the scenario depicted

(Force) x (Time of action) = (Mass) x (Increase in speed)

But our relativistic reasoning has just shown us that it depends on the initial speed

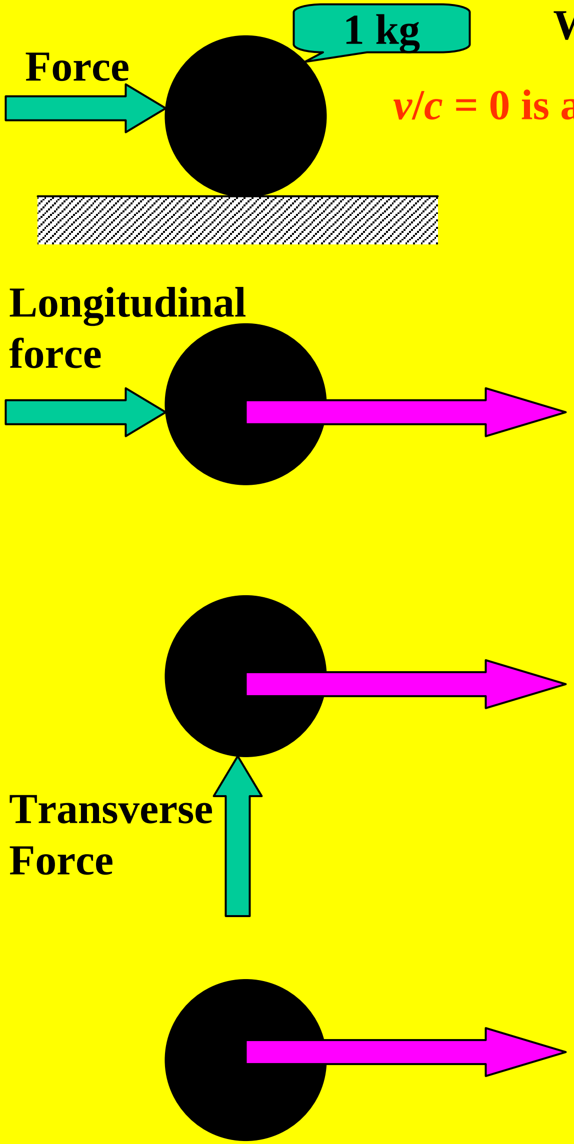
(Force) x (Time of action) = (Mass) x Increase in[(Speed factor) x (speed)]

The Speed factor (in this relationship) is the now familiar γ and is = 1 when speeds are small and increases indefinitely towards infinity as speeds approach the speed of light when *V/c* = 1

$$= \frac{1}{\sqrt{1-(V/c)^2}}$$

BUT RELATIVISTIC MASS IS A STRANGE 'ANIMAL' AS YOU WILL SEE ON THE NEXT SLIDE

We are using the symbol m_r to mean relativistic mass.



$v/c = 0$ **is at rest**

$\vec{\text{Force}} = m \times \vec{(\text{acceleration})}$ $m = 1$ kg

$v/c = 0.866$

$\vec{\text{Force}} = m_r \times \vec{(\text{acceleration})}$ $m_r = 8$ kg

$$m_r = \frac{m}{\left[\sqrt{1-(V/c)^2}\right]^3}$$

$v/c = 0.866$

$\text{Force}\uparrow = m_r \times (\text{acceleration})\uparrow$ $m_r = 4$ kg

$$m_r = \frac{m}{\left[\sqrt{1-(V/c)^2}\right]^2} = \frac{m}{1-(V/c)^2}$$

$v/c = 0.866$

Momentum = m_r x (velocity) $m_r = 2$ kg

$$m_r = \frac{m}{\left[\sqrt{1-(V/c)^2}\right]}$$

So the relativistic mass of this 1 kg object can be 1, 2, 4 or 8 kg !!

ONLY IF YOU WANT TO KNOW

MASS IS A VERY STRANGE ANIMAL

Below is a cutting from Einstein's June 1905 paper 'ON THE ELECTRODYNAMICS OF MOVING BODIES' (p919 of the German text) which appeared in the book 'THE PRINCIPLE OF RELATIVITY' (Methuen 1923)

Einstein uses μ for (rest) mass and V for the speed of light.

Mass x Acceleration = Force

Massenzahl × Beschleunigungszahl = Kraftzahl

aufrechterhalten, und wenn wir ferner festsetzen, daß die Beschleunigungen im ruhenden System K gemessen werden sollen, so erhalten wir aus obigen Gleichungen:

Relativistic masses

$$\text{Longitudinale Masse} = \frac{\mu}{\left(\sqrt{1-\left(\frac{v}{V}\right)^2}\right)^3},$$

When the force is in the direction of existing motion

$$\text{Transversale Masse} = \frac{\mu}{1-\left(\frac{v}{V}\right)^2}.$$

When the force is transverse to the existing motion

Natürlich würde man bei anderer Definitio

As shown on the previous slide.

In the previous example it should be observed that in both cases the same force is being applied for 1 *s* on *B*'s stopwatch but *A* will observe the force to have been applied for different time intervals in the two cases.

Low Speed

When ***V/c* = 0.2 to 0.3848 (an average value of 0.27)** then ***A*** observes the force to have been applied for about

$$t_A \cong \frac{1+(V/c)}{\sqrt{1-(V/c)^2}} = \frac{1+0.27}{\sqrt{1-0.27^2}} = 1.32\ s$$

High Speed

When ***V/c* = 0.8 to 0.862 (an average value of 0.83)** then ***A*** observes the force to have been applied for about

$$t_A \cong \frac{1+(V/c)}{\sqrt{1-(V/c)^2}} = \frac{1+0.83}{\sqrt{1-0.83^2}} = 3.3\ s$$

So in the high speed case for *A* the increase in *V/c* is only 0.062 even though he sees the force to have been applied for more than twice as long as the low speed case when the speed increase was three times as much (*V/c* = 0.1848).

The reason for the difference between Newton’s and Einstein’s results lies entirely with the fact that Newton was not taking account of the fact that information signals take time to travel.

Römer, observing the increasing eclipse times of Jupiter’s moon Io in 1676 (when Newton was 34) as Jupiter accelerated away from Earth. He concluded that the reason was the increasing distance the images were having to travel as Jupiter increasingly receded from the Earth (i.e. so the Doppler effect was discovered nearly two centuries before Doppler 1824), so it was known that light took time to travel but the true significance of this eluded everyone until Einstein’s 1905 paper. Römer estimated that the images must travel at a speed in excess of 225,000 *km/s*.

Even if Newton had considered the matter he could not have drawn the conclusions that Einstein drew, because there would be no way he could know that all observers, observe light to travel at the same speed (which is an observed fact and fundamental to the reasoning involved) because time could not be measured with anything like the accuracy necessary to determine this fact.

All signals experience time delays since they travel at a maximum speed of c = 186,000 miles per second (300,000 *km/s*). The accepted value of c was standardised in 1975 as c = 299,792,458 *m/s* .

Subsequent slides illustrate these issues graphically.

ONLY IF YOU WANT TO KNOW

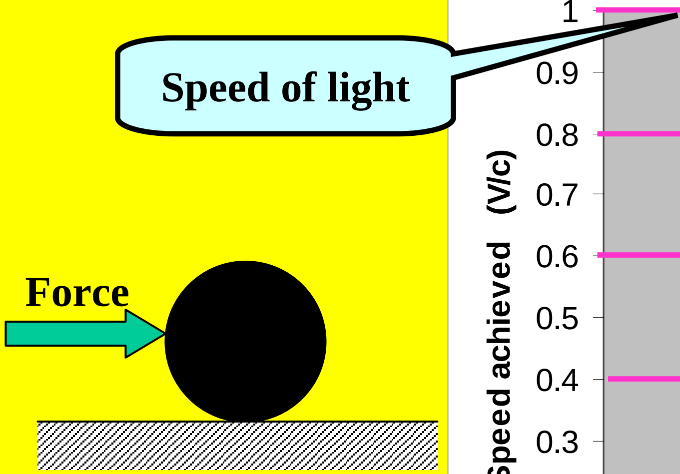
SPEED OF LIGHT

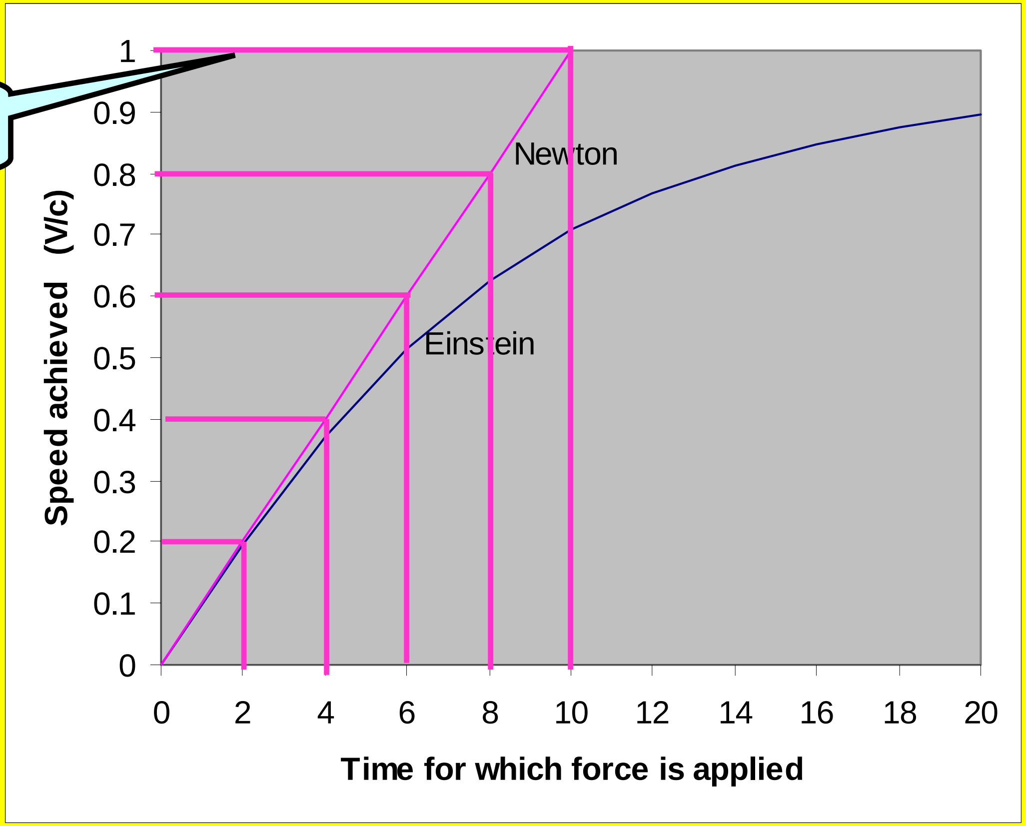
Romer, Olaf Christensen	(1676)	Danish astronomer	
	Eclipses of Jupiter's moon Io. Estimated		$c > 225 \times 10^5$ km/s
Bradley, James	(1727)	English astronomer	
	Stellar abberation estimated		$c \approx 284 \times 10^5$ km/s
Fresnel, Augustin	(1814-22)	French engineer	
	Inventor of interferometry		
Fizeau, Armand	(1849)	French physicists	
	Tried to detect the ether using toothed wheel method but failed		
Foucault, Jean with Fizeau, Armand	(1850)	French physicists	
	Accurate light speed within ± 1%		
Michelson, Albert	(1881)		
	Developed apparatus to detect ether at Berlin in 1881		
Michelson, Albert with	(1887)		
	Edward Morley refined the apparatus to very high accuracy at Ohio		
Michelson, Albert with Morley, Edward	(1924-26)		
	Rotating mirrors with 70 km base line		$c = 299.796 \times 10^5$ km/s
Michelson, Pease & Pearson	(1932)	American physicists	
	1 mile evacuated tunnel		$c = 2.99774 \times 10^5$ km/s
Froome	(1958)		
	Using interferometry		$c = 2.997925 \times 10^5$ km/s

The graph below shows the speed achieved when a given force is applied to an object for 2 units of time, repeatedly.

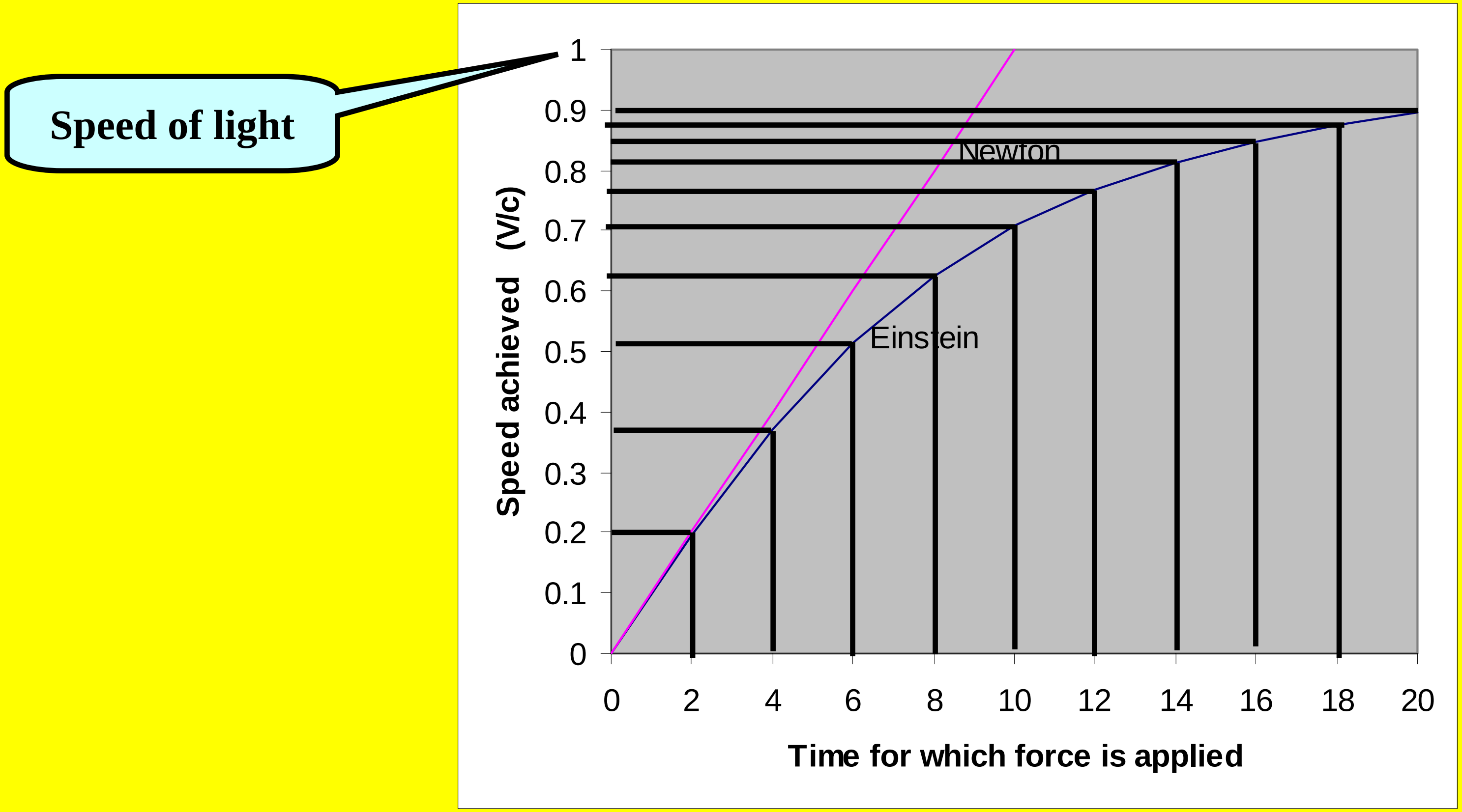
Newton tells us that each time the force is applied it will produce the same increase in speed.

Newton is saying that if (in this example) we apply the force for 10 units of time then the object will reach the speed of light and if continued thereafter will exceed the speed of light.





As we have seen, our reasoning (due to Einstein) has shown that each time the force is applied the speed increase gets less and less (as shown below). This is to say that its resistance to having its speed increased - *inertia* - becomes greater and greater as the speed increases.



We would have to apply the force for ever for the object to reach the speed of light.
In each time step an increasing distance is moved and so as the speed of light is approached the force moves through an ever increasing total distance. Force times distance moved through, is the definition of energy. So to reach the speed of light would require an ever increasing amount of energy - an infinite amount. No such amount of energy exists. An object of finite mass then cannot be accelerated to the speed of light.

However this does not mean that something of indefinitely small (rest) mass cannot be accelerated to a speed indefinitely close the the speed of light because as we have seen

$$\textbf{(Force) x (Time of action) = (Mass) x } \frac{1}{\sqrt{1-(V/c)^2}} \textbf{ x (Increase in speed)}$$

If for instance an electron, Mass = 9 x 10 $^{-31}$ ***kg*** **is accelerated from rest to a speed within 0.01% of the speed of light corresponding to** ***V/c*** **= 0.9999**

then $\frac{1}{\sqrt{1-(V/c)^2}}$ **= 70 and** ***V*** **= (3 x 10**^{8}**) x 0.9999 = 3 x 10**^{8} ***m/s*** **(effectively) so**

$$\textbf{(Force) x (Time of action)} = (9 \times 10^{-31}) \times 70 \times (3 \times 10^{8}) = 1.89 \times 10^{-20} \; Ns$$

Which would be achieved by applying a force of

$$\frac{1.89}{100{,}000{,}000{,}000{,}000{,}000{,}000} \quad \textbf{Newtons for 1 second}$$

Photons (particles of light) move much closer to 3 x 10 ^{8} ***m/s*** **and would have a much smaller (rest) mass than the electron however a photon's rest mass is a meaningless concept because they are never observed to be at rest.**

Newton's law of motion is

(Force) = (Mass) x (Acceleration)

i.e. $$F = m \times a$$

But as we saw, special relativity requires some modification because it takes into account the time it takes for information signals to travel, then

(Force) = (Mass) x (Speed factor) x (Acceleration)

We can either leave this in this form OR write it as

(Force) = [Mass x (Speed Factor)] x (Acceleration)

and call [Mass x (Speed Factor)], '<u>Relativistic Mass</u>' and denote it by m_r

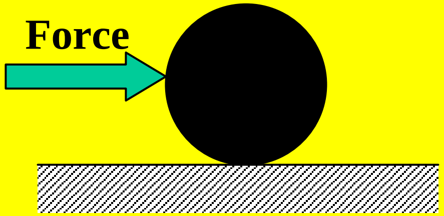
whence (Force) = [Relativistic Mass] x (Acceleration)

i.e $$F = m_r \times a$$

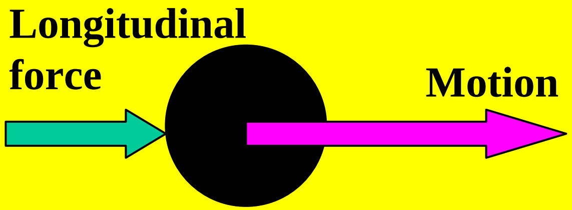
thus retaining the form of Newton's law but, it would be more helpful if m_r was called Inertia and denoted by a different symbol (say I).

So when we look at the variation of relativistic mass (i.e. inertia) we are really just looking at the variation in the Speed Factor. But as illustrated earlier, this speed factor is different according to which direction the force is being applied and whether it is being used in the above force form of Newton's law or in the momentum form (Force) x (Time) = (Mass) x (Increase in Velocity).

As we saw earlier speed factor (relativistic mass) is different in different situations



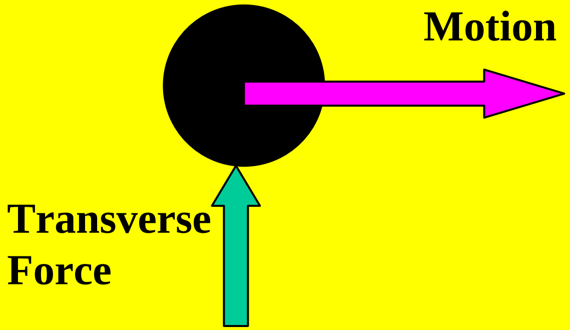
Speed factor = 1 at low speeds.



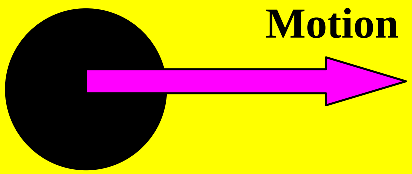
More generally in the relationship

Force = Mass x Speed Factor x Acceleration.

$$\text{Speed factor} = \frac{1}{\left[\sqrt{1-(V/c)^2}\right]^3}$$



$$\text{Speed factor} = \frac{1}{\left[\sqrt{1-(V/c)^2}\right]^2}$$



In the relationship

Momentum = Mass x Speed Factor x Velocity

$$\text{Speed factor} = \frac{1}{\left[\sqrt{1-(V/c)^2}\right]}$$

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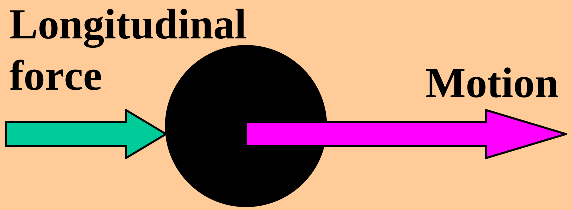
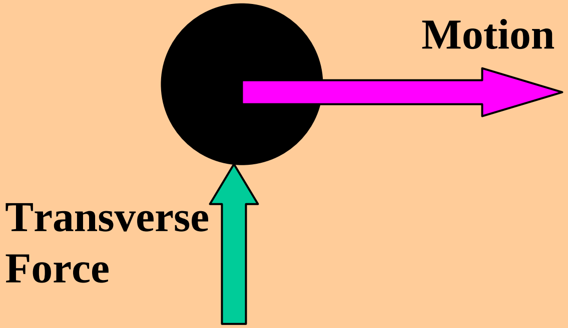
Einstein puts it similarly but talks not about speed factor but about longitudinal and transverse (relativistic) mass m_r = m x *(speed factor)* i.e.

Force = m_r x acceleration.



Longitudinal force $= \dfrac{m}{\left[\sqrt{1-(V/c)^2}\right]^3}$ **x acceleration**



Transverse force $= \dfrac{m}{\left[\sqrt{1-(V/c)^2}\right]^2}$ **x acceleration**

These equations have been obtained using logical deduction by application of Newton's laws of motion taking into account that signals take time to travel.

The problem with them is that they are not compatible with the conservation of momentum

Force x (time of action) $= \dfrac{m}{\sqrt{1-(V/c)^2}}$ **x (increase in velocity)**

which is regarded as a universal over-riding law of nature.

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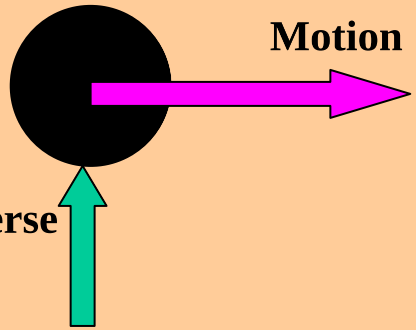
The transverse equation of motion which is compatible with conservation of momentum is NOT



Transverse force = $\dfrac{m}{\left[\sqrt{1-(V/c)^2}\right]^2}$ **x acceleration** **(1)**

BUT

Transverse force = $\dfrac{m}{\left[\sqrt{1-(V/c)^2}\right]}$ **x acceleration** **(2)**

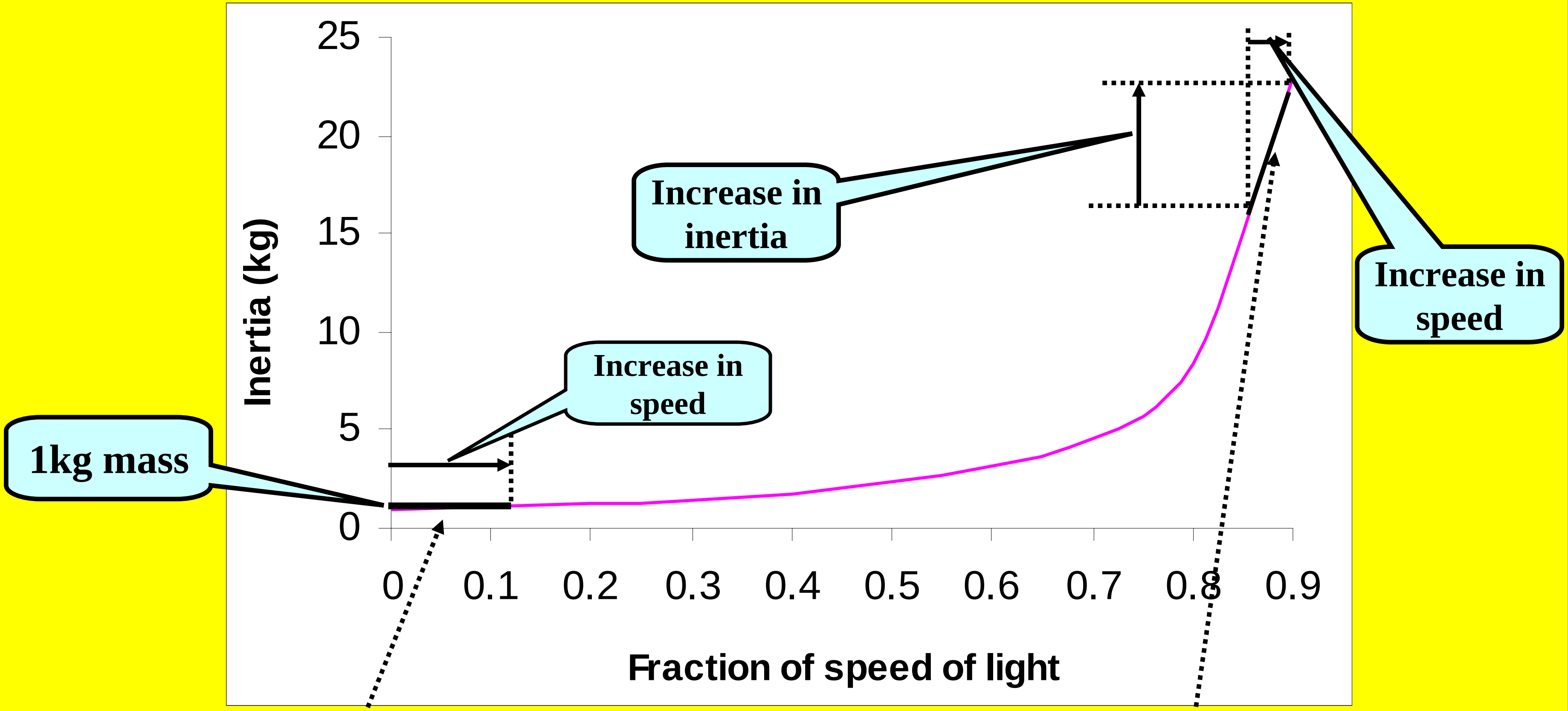
Only A.P.French in his book 'Special Relativity' (p137) seems to draw attention to this salient fact. His resolution to this incompatibility is however derisory. He suggests that equation (1) be used to obtain the value of the necessary transverse force then to multiply the result by the factor

$$\sqrt{1-(V/c)^2}$$

thus obtaining the value you would have got if equation (2) had been used.

The incompatibility issue (or paradox) is not raised by Einstein in his paper.

We see the following variation in inertia of a 1 kg mass as its speed is increased in the case where the force is applied in the direction of the motion, as below. In this case the speed factor is as stated earlier and is given by the expression $1\Big/\left(\sqrt{1-(V/c)^2}\right)^3$



At low speeds the effect of applying a force to the mass is primarily to increase its speed, its increase in inertia being very small.

At high speeds the effect of applying a force to the mass is primarily to increase its inertia, i.e. its increase in speed becomes smaller and smaller as the speed increases.

Because photons have inertia, force applied to them changes either their inertia or their direction of motion. It cannot increase their speed because they are already traveling at the speed of light and have indefinitely large inertia in the direction of motion.

Who would have thought that the mere observation that information takes time to travel would make such a profound effect on our understanding of the way matter behaves.

STUDY No 3 : LENGTH

Question: When is a 1 metre rod not a 1 metre rod ?

Answer: When it's moving.

This issue requires a consideration of what we are actually observing when we observe length.

The action of observing with the eyes the length of an object when it's in motion is made difficult by the fact that it's blurred due to the fact that the retinal and visual cortex responses are not fast enough to retain a clear image.

What we actually see is a series of images, each created by photons arriving at the retina *all at the same instant of time*.

If we blinked very fast, one clear image should be visible, but our visual system is not able to process an image of such short duration.

What we need is a camera with a very fast shutter action and a photographic emulsion which is fast enough to capture the image.

From our previous results

Where $\beta = V_{BA}/c$ & $\gamma = 1/\sqrt{(1-\beta^2)}$

Definition of relativistic velocity

$$V = \frac{x}{t - x/c}$$

Relativistic addition of velocities for *A* and *B* when viewing an object *C*

$$V_A = (V_B + \beta) \times \left(\frac{1}{1 + V_B/c \times \beta} \right)$$

Relationship between times for *A* and *B*

$$t_A = \gamma(1+\beta) \times t_B$$

From which with some tedious algebraic re-arrangement we can extract x_A

$$x_A = \gamma\left[(1-\beta)x_B + \beta c\, t_B\right]$$

Where, with images arriving at *A* simultaneously, $t_A = 0$ so $t_B = 0$

$$x_A = \gamma\left[(1-\beta)x_B\right]$$

so lengths x_A observed by *A* are related to lengths x_B observed by *B* by this relationship

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$$x_A = \gamma\left[(1-\beta)x_B + \beta\, c\, t_B\right]$$

This relationship is part of what is known as the Lorentz transform

If we substitute our relationship $t_B = \tau_B + \dfrac{x_B}{c}$

between *B*'s observed time t_B and his event time τ_B into it we obtain

$$x_A = \gamma\left[x_B + \beta\, c\, \tau_B\right]$$

Which is how you will invariably see it in text books where τ is event time NOT clock time.

If moving in the line-of-sight, the length x_A of a rod shown on the *A*'s photograph is given by the relationship as just deduced where x_B is the length *B*'s photograph shows it to be

$$x_A = \gamma(1-\beta)x_B = \frac{1}{(1+\beta)}\frac{1}{\gamma}x_B$$

It shows that *A's* photograph shows *B*'s rod as shortened when *B* is in recession ($\beta > 0$) but lengthened if *B* is in approach ($\beta < 0$). These are the lengths (x_A) that *A* would actually see on his high speed photograph BUT it is NOT what is meant by length in relativity.

The relativistic length is defined as

$$x_A = \sqrt{1-(V/c)^2}\,x_B = \frac{1}{\gamma}x_B$$

Which is longer than the photographic length when in recession ($\beta > 1$) but shorter if in approach ($\beta < 1$).

It accounts only for the effect of time dilatation and ignores the Doppler effect. It says that *A*'s relativistic length is shorter than x_B whether the rod is in recession or in approach and is NOT what your photograph will show but this value will be the same for everyone in *A*'s group of observers whereas the photograph will show a different length for individual observers in his group and so would not be much use when discussing the length of the rod.

Roger Penrose in his book 'The Emperor's New Mind' is one of the few texts to point this out.

The next slides illustrate these phenomena.

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If we put t_B from our previous time relationship $t_A = \gamma(1+\beta)\times t_B$ **into our previous distance relationship**

$$x_A = \gamma[(1-\beta)x_B + \beta c t_B]$$

we obtain

$$x_A - ct_A \frac{\beta}{1+\beta} = \gamma(1-\beta)x_B$$

In the photographic case the images arrive together, $t_A = 0$ so as before we obtain

$$x_A = \gamma(1-\beta)x_B = \frac{x_B}{\gamma(1+\beta)}$$

The relativistic length is the estimated difference between the location of the two ends of the rod on *A*'s tape at the same instant of time on his stopwatch. Which is to say, where these two ends are at the same instant of *A*'s event time i.e.

$$\tau_A = t_A - x_A/c = 0 \quad \text{that is when} \quad t_A = x_A/c$$

If we put this in our distance relationship above then with a little algebraic re-arrangement we obtain our relativistic length estimate

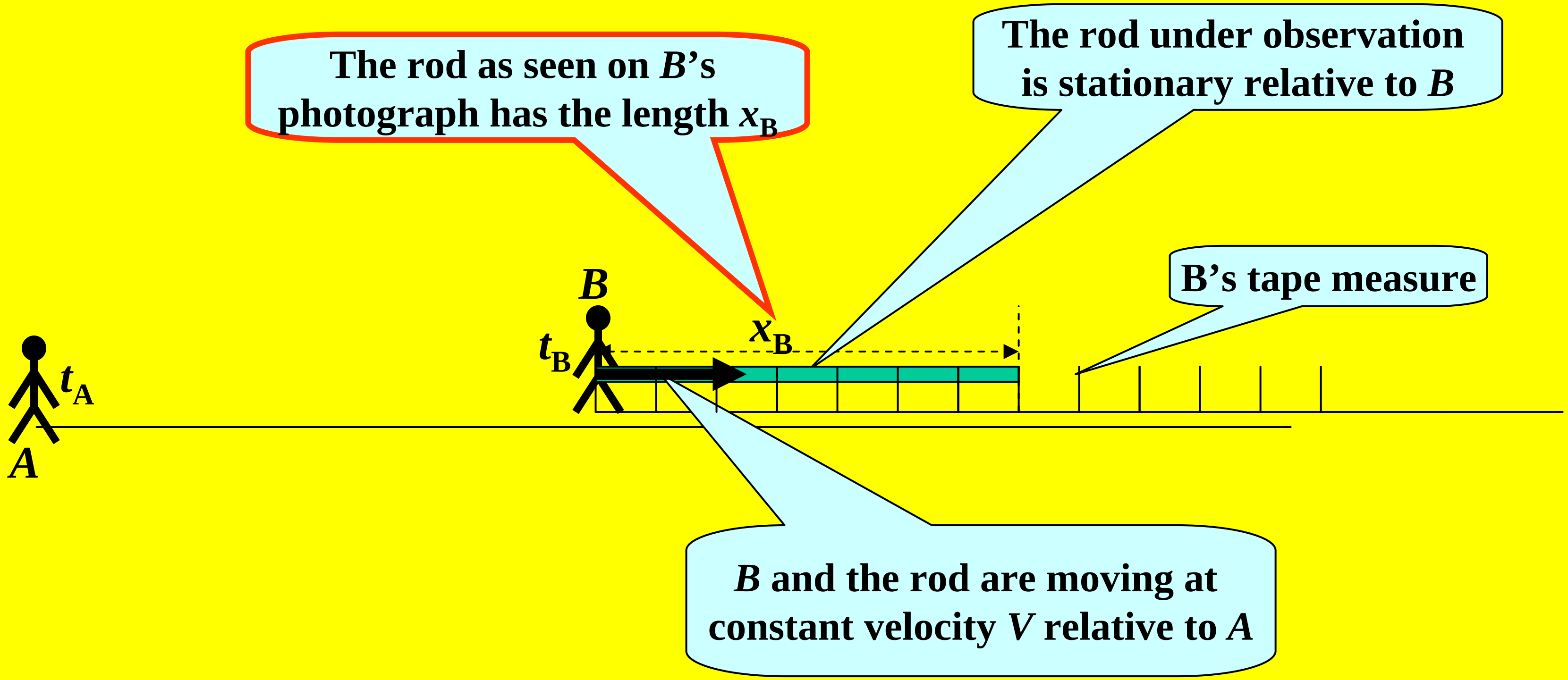
$$x_A = \frac{1}{\gamma}x_B$$

What the symbols on the next slides refer to

x **is observed distances**

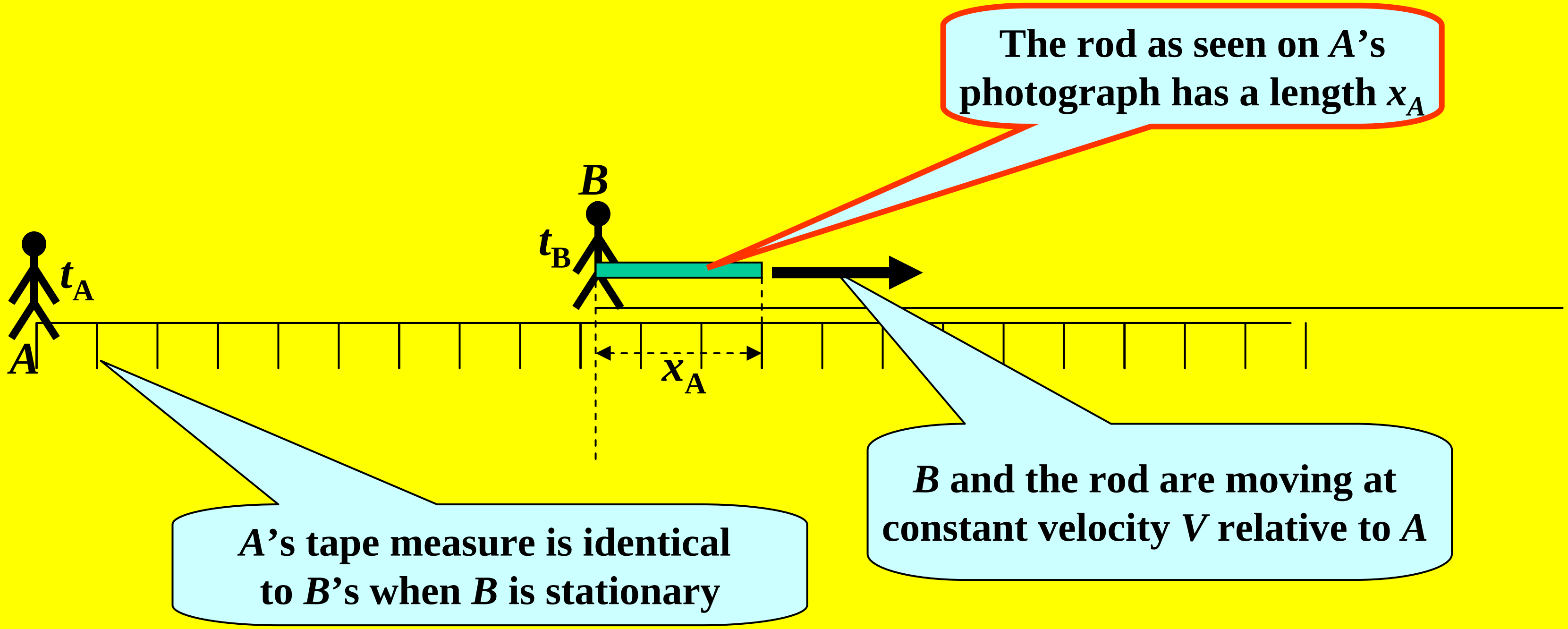
t **is observed times**

observers *A* and *B*



In the examples which follow, *B* is receding from (or approaching) *A* directly (meaning in line of sight) as in the illustration below.

Of course if the rod is moving in-line-of-sight (as below) the tapes would need flags to indicate the tape values at each end of the rod because we would only see the rod in end view.



This length (x_A) is what will appear (i.e. be observed) on *A*'s photograph

$$x_A = \gamma(1-\beta)x_B = \frac{1}{\gamma(1+\beta)}x_B$$

So if x_B refers to the length of *B's* 1 *m* rod then if he is receding from *A* at 86.6 % of the speed of light (i.e. β = 0.866) then $\gamma = 1\big/\sqrt{1-\beta^2} = 2$ so *A*'s photograph will show the length of the rod to be shorter

$$x_A = 2\times(1-0.866) = 0.268\ m$$

But if *B* is approaching *A* at 86.6 % of the speed of light (i.e. β = - 0.866) then γ = 2 so *A*'s photograph will show the length of the rod to be longer

$$x_A = 2\times(1-(-0.866)) = 3.73\ m$$

But the relativistic length which ignores the Doppler effect is defined as

$$x_A = \frac{1}{\gamma}x_B = 0.5\ m$$

so in the above case, whether *B* is in approach or recession to *A*, γ = 2 so for *A* the relativistic length of *B*'s 1 *m* rod is 0.5 *m* and is always shorter if the rod is in motion. This is the separation that *A* estimates the two ends of B's rod to be on his (*A*'s) tape at the same instant of time on his stopwatch. But the photograph shows the separation of the two ends of the rod when the images arrive to the camera at the same time on his stopwatch. For the latter to happen the image from the far end of *B*'s rod must have left earlier than the image from the near end so by now it will be further away. Hence the difference.

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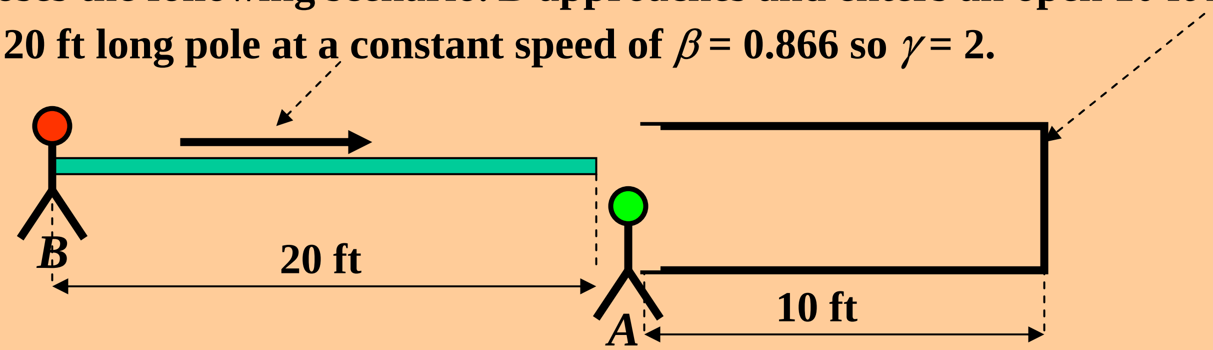
THE GARAGE PARADOX

This poses the following scenario. *B* approaches and enters an open 10 ft long garage with a 20 ft long pole at a constant speed of $\beta = 0.866$ so $\gamma = 2$.



***A* is the garage door keeper. Will the pole fit into the garage so that *A* can close the garage door ?**

Apparently NOT.

We need to be clear about the information we are being given.
For *A* the two ends of the garage are separated by 10 ft.
For *B* the two ends of the pole are separated by 20 ft.

***A* estimates that at any instant of time the two ends of the rod are separated by a distance of $20/\gamma = 20/2 = 10$ ft so when one end of the rod is in contact with the far end of the garage the other end, where *B* is, is at the garage door where *A* is standing. The rod just fits into the garage whilst retaining its integrity. The door could be shut with the pole inside.**

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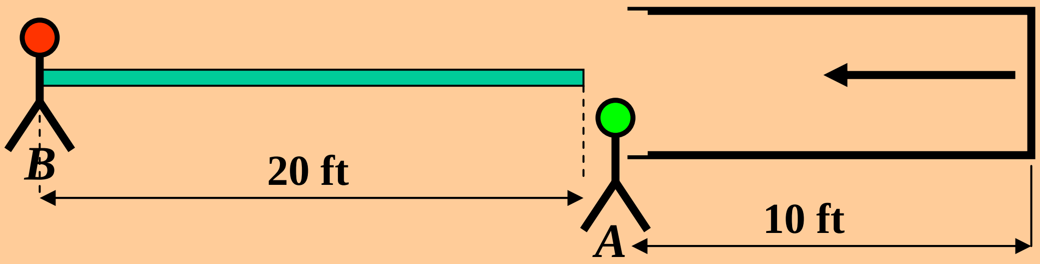
If we consider the scenario from *B*'s point of view, the garage approaches him continuously at $\beta = -0.866$ so again so $\gamma = 2$.



B **estimates that at any instant of time the two ends of the garage are separated by a distance of $10/\gamma = 10/2 = 5$ ft so when one end of the rod is in contact with the far end of the garage the rod has 15 ft yet to enter. So the rod does not fit into the garage <u>whilst retaining its integrity</u>.**

THE PARADOX IS THAT WHILE *A* CONCLUDES THAT THE ROD WILL FIT INTO THE GARAGE <u>WITH ITS INTEGRITY IN TACT</u>, *B* CONCLUDES THAT IT WILL NOT.

THEY CAN'T BOTH BE TRUE !

Rindler, W. 'Introduction to Special Relativity' second edition. *Oxford Science Publications* (p29) argues that *B* can enter the rod into the garage because its end wall consists of a large concrete block so that when the end of his rod is in contact with it, the rod will be crushed but that this effect will not affect him at the near end because it will take the compression wave much longer to travel the 5 ft length of rod now in the garage than it takes for his end of the rod to travel the outstanding 15 ft when $\beta = 0.866$!!! SEEMS TO MISS THE POINT.

You may have read or heard that just as moving clocks run slow, moving rods get shorter.

This is to say that the estimated separation of the two ends of the rod at any instant of the observers time is less if the observer sees it in motion than when he sees it stationary.

But a photograph will show that while this is true if the rod is in recession it will look longer if in approach. It is true that *A*'s estimate of the relativistic length is always shorter but in neither case is this what he will actually see. Relativistic length is not something we can actually see, just as relativistic time is not something we can observe on a clock but the relativistic values are the same for everyone in our group whereas the observed values will be individual to each group member and not therefore very useful as a measure.

The term group implies anyone who is stationary relative to us. Texts refer to this group as all being in the same 'reference frame'.

STUDY No 4 : ENERGY

From Newton's law of motion we are able to calculate the energy *E* required to accelerate a mass *m* from rest to a speed of *V* as

$$E = m\frac{1}{2}V^2 \quad = mc^2\,\frac{1}{2}\left(\frac{V}{c}\right)^2$$

which is what we use everyday and works fine for speeds where *V* is much less than *c*.

Working from Newton's law we can show that the relativistic form of the force-acceleration relationship results in the following expression for the kinetic energy

$$E = mc^2\left[\frac{1}{\sqrt{1-(V/c)^2}} - 1\right]$$

As written, both depend on *V/c*, the speed of the mass *V* relative to the speed of light *c* but in quite a different way.

For everyday applications these provide the same value e.g. for a rocket leaving the earth at escape speed of 12 *km/s* the difference between these two values is

0.00000012%

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HOW WE DEDUCE THE EQUATION FOR KINETIC ENERGY

Kinetic Energy (*E*) is defined as the (applied force) x (the distance moved) i.e *E* =*F* x *x*

Using the relativistic form of the force, acceleration relationship $F = m\gamma^3 \dfrac{dV}{d\tau}$

and the fact that $V \equiv \dfrac{dx}{d\tau} \equiv \beta c$ **then by algebraic substitution and some re-arrangement we obtain for a small displacement *dx***

$$dE \equiv F dx = m\gamma^3 V dV = m\gamma^3 c^2 \beta d\beta = mc^2 d\gamma$$

which by simple integration yields $E = mc^2 \left[\gamma\right]_{rest\ when\ \gamma=1}^{\gamma\ when\ moving\ at\ speed V} = mc^2\left[\gamma - 1\right]$

where $\gamma \equiv \dfrac{1}{\sqrt{1-(V/c)^2}}$ **hence**

$$E \equiv mc^2\left[\frac{1}{\sqrt{1-(V/c)^2}} - 1\right]$$

For something moving at or near the speed light $\gamma \to \infty$ **so it is sufficient to say** $E = mc^2\gamma$ **since there is no point in subtracting 1 from** ∞ **.**

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The following is a cutting from Einstein's paper 'On the Electrodynamics of Moving Bodies' (June 1905) which is what is referred to as his theory of special relativity. Here W is being used for the kinetic energy which is the Work done by the force and which is the definition of the kinetic energy

mind that during the whole process of motion which we are considering, the first of the equations **(A)** applies, we therefore obtain

$$\begin{aligned} W &= \int \epsilon X\, dx = m \int_0^v \beta^3 v\, dv \\ &= mc^2 \left\{ \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right\}. \end{aligned}$$

where *m* is the quantitative measure of the mass of the body (as by weighing) as initially introduced in this presentation. Nowhere does he introduce the term relativistic mass

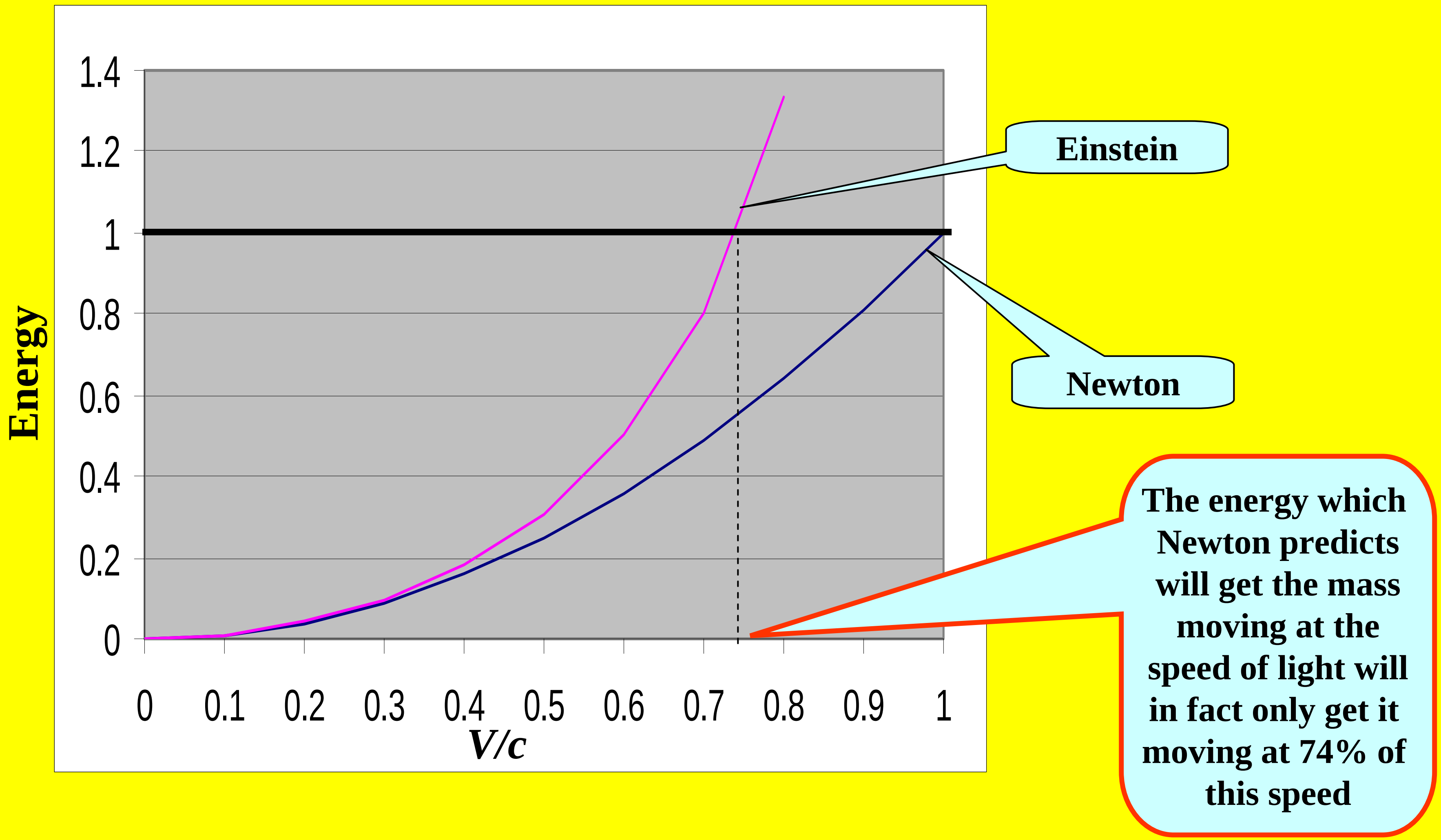
$$m_r \equiv m\gamma$$

and nowhere does he state that $E = mc^2$

If we introduce the concept of relativistic mass then the above equation will appear as

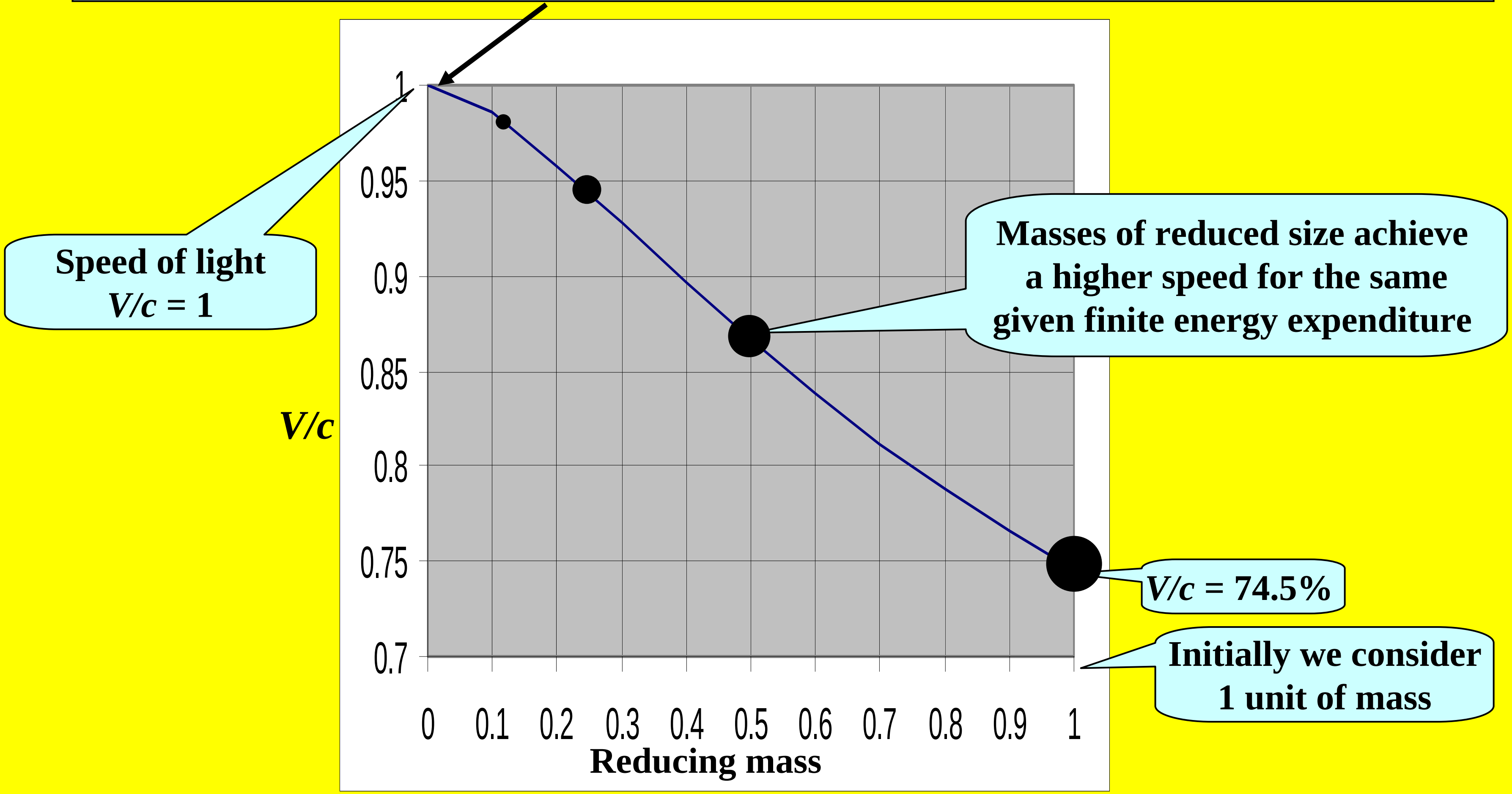
$$E = c^2 \{m_r - m\}$$

The graph below shows the energy required to get a mass moving at speeds of *V/c*. We are calling the energy that Newton says we would need to get the mass moving at the speed of light (*V/C* = 1) as 1 unit of energy



The graph below shows the speed achieved for a given finite energy expenditure applied to less and less mass. The amount of energy under consideration has been chosen arbitrarily as that required to get unit mass moving at 74.5% of the speed of light

A mass of indefinitely small size can be accelerated to a speed indefinitely close to the speed of light (*V/c* = 1) and can have a finite amount of kinetic energy (e.g. a photon).



From the equation of motion we can deduce that to accelerate a mass *m* from rest to a speed corresponding to *V/c*, the kinetic energy *E* (which is the force times the distance the mass has moved) is given by the equation

$$E = m\left[\frac{1}{\sqrt{1-(V/c)^2}} - 1\right]c^2$$

As *V/c* approaches a value of one (i.e.the speed of light).

$$\frac{1}{\sqrt{1-(V/c)^2}} \Rightarrow \infty$$

In this case there is no point in subtracting 1 from a value indefinitely large so we might just as well ignore it and write the above as

$$E = m\left[\frac{1}{\sqrt{1-(V/c)^2}}\right]c^2$$

We call the value $m\left[\frac{1}{\sqrt{1-(V/c)^2}}\right]$ **the relativistic mass m_r so the top equation can be written as**

$$E = m_r c^2$$

Since c^2 has a fixed value (9 x 10^{16} m^2/s^2) this equation effectively says we can equate energy to mass or *vice versa* i.e. it is a matter of semantics as whether one chooses to call it energy or mass.

But note that the mass in $E = m\,c^2$ is the relativistic mass m_r and does not mean the quantity of matter as you might have thought.

When an object radiates energy as photons (light 'particles') they are moving at the speed of light '*c*' so as just illustrated they have an energy of

$$E = m_r c^2$$

Their mass *m* may be indefinitely small but at speed *c* their inertia factor $\frac{1}{\sqrt{1-(V/c)^2}}$ **is indefinitely large and their product (relativistic mass)**

$$m_r = \frac{m}{\sqrt{1-(V/c)^2}}$$

and is finite just as e.g.

$$\frac{1}{1{,}000{,}000{,}000} \times 2{,}000{,}000{,}000 = 2$$ **is finite**

So their energy $m_r c^2$ is finite since $c^2 = 9 \times 10^{16}$ $(m/s)^2$ is finite.

The mass *m* is their mass when at rest, but since photons are observed always to move at the speed = *c*, it is meaningless to speak of their mass when at rest.

The conservation of energy (like the conservation of momentum) is regarded as an overriding principle of nature. If then a mass can be converted entirely into radiation, that radiation contains a total energy $m_r c^2$ and the conservation of energy then predicts that the original mass must have contained that amount of energy.
A relationship between the kinetic energy (*E*) of light particles (or ether matter) and their (*mass* x c^2) long preceded the relationship attributed to Einstein. Such a relationship had been propounded by Samuel Tolver Preston in 1875, by Poincare in 1900 and by Fritz Hasenohrl in 1904.

Note that the equation form $E = mc^2$ on the previous slide does not appear in either of Einstein's 1905 two papers which are collectively referred to as his theory of relativity

On the Electrodynamics of Moving Bodies ***June 30 1905***

Does the Inertia of a Body depend upon its Energy-Content ? ***September 27 1905***

(these are readily available on the Internet in German and English translation)

In his second paper he purports to show that when a body emits energy in the form of radiation the body loses *Inertia,* then in the paper he reverts to calling it *mass* and assumes that the body loses mass.

In the latter paper he merely demonstrates that the kinetic energy denoted by K (where on a previous slide we used E) of a mass m moving at speed V is given by

$$K - K_0 = m\left[\frac{1}{\sqrt{1-(V/c)^2}} - 1\right]c^2$$

where K_0 is its kinetic energy when at rest.

As is pointed out in *Herbert Ives* didactic analytical paper

Derivation of the Mass-Energy Relation ***Feb 28 1952***

Einstein's reasoning which was based only on the conservation of energy, is false. Einstein's paper ends simply by assuming that which he purports to be proving. The correct proof necessarily requires also the application of the conservation of momentum.

Einstein neither proved nor stated that $E = mc^2$.

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