“Shedding Light on Einstein’s Special Theory of Relativity (1905)

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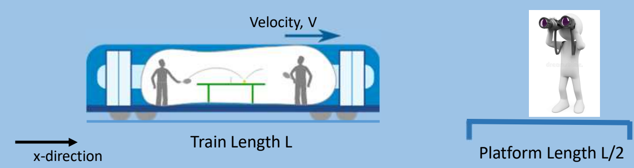
- In 1905 the German-born **theoretical physicist** Albert Einstein proposed his ‘Theory of Special Relativity’ which was to completely change the direction of modern physics. Einstein published 4 scientific papers in 1905, as well as submitting his PhD thesis. Two were about SR, but it was the paper on the photoelectric effect which won him the Nobel prize in 1921. 1905: working at Patent Office in Berne, Switzerland not as an academic at that time
- But what is a theory of relativity? Relativity is the study of **space, time & motion**, and the relationships between them. Einstein formulated the principles of the theory (using mathematical analysis, thought experiments, and imagination). Of course any theory in science has to be verified by repeatable experiments, and SR theory has been found to be correct in all cases. The previous theory proposed by Galileo in the 17th century was found to be confined to low speed motion.
- Einstein’s new theory is called ‘special’ because it is limited in extent: it does not include accelerations & gravity. His ‘general theory’ introduced 10 years later remedies this shortcoming, & becomes his ‘theory of gravitation’.
- In this talk I will concentrate on some of the phenomena which are predicted by SR theory, but also add some material about some scientists involved in it’s development. Also some material about SR theory’s successor: GR theory, published by Einstein in 1915. Includes gravity & accelerations
- Uses: particle physics (eg at Cern LHR), GPS systems, astronomy (eg study of black holes)

- Brownian motion: the erratic random movement of **microscopic particles** in a fluid, as a result of continuous bombardment from **molecules** of the surrounding medium.

Definition of Relativity (Oxford English Dictionary)

Relativity is the dependence of various physical phenomena on the relative motion of an **observer** and an **observed object**, (especially regarding the nature and behaviour of light, space, time and gravity).



Formal definition of relativity.
Images below show relative motion between an **observer** on a platform on the right, and the observed **object** (the train). Each has its own **frame of reference**.
Think about relativity **pre-Einstein:**

(a) Imagine the train to be stationary with a game of table tennis in progress. How would this change if the train is moving in a straight line on a smooth track
at 100 km/hour with no acceleration? Would the players be aware of the change of velocity? They may notice the surroundings rushing past the windows
(at -100 km/hour) but their game would not change at all. In the frame of reference of the train.
(b) Basket ball player & bounce of ball
(c) In frame of train passengers have relative velocity of zero. If another passenger walks up the train at 3km/h and the train is travelling at 100km/h with reference
the stationary observer will see the passenger travelling at 103km/h (addition of the velocities). And if the observer now runs towards the train at 6km/h then
The total relative velocity between train and observer will be 94km/h

Post-Einstein:
A foretaste of what SR predicts. Train has length L, and platform has length L/2. Find the situation in which the train fits the platform. If train velocity v = 0.9 x speed of light
& ignoring practicalities which arise, the length of the train will have reduced by a factor of 2 when passing platform. Counter-intuitive. This is hard to accept but has been proved by experimenting with atomic particles travelling at relativistic speeds. Trains often used by Einstein to illustrate aspects of SR.
This a hypothetical example of SR and it's prediction of **Length Contraction** in moving bodies. Length reduces with velocity

Length contraction **example:** Concorde contracts 1.30nm at Mach 2

Theories of Relativity

- Galileo's Principle of Relativity(1632)
- Newton's endorsement (C18th)
  (also introduced *3 Laws of Motion, & Law of Gravity*)
- Einstein's Special Theory (1905)
- Einstein's General Theory (1915)
  (*a Theory of Gravitation*)

$$F = \frac{GmM}{r^2}$$

People that have made very significant contributions to the subject of relativity.

Galileo: An Italian Renaissance mathematician & astronomer. Introduced the Principle of Relativity. Mainly based on intuition, observation & common sense.
Unsuccessfully attempted to measure the speed of light. Studied gravity in detail using inclined planes.
Newton: He agreed with Galileo in connection with relativity. Most famous for introducing his 3 Laws of Motion Force/F=ma/action-reaction.
Also introduced Universal Law of Gravity to study the solar system, see side equation
Einstein: With SR and GR showed that Newton's contributions needed modifications for bodies moving at high speeds.
Also GR replaced Newton's Law of Gravity with a new Theory of Gravity. Gravity now the curvature of spacetime, a new integrated description of space and time.

$$3.0 \times 10^8$$

This is the most important number in Einstein's theories, and this very sparse slide emphasises this. Three hundred million or 3 times 10^8, not an exact number but very close.

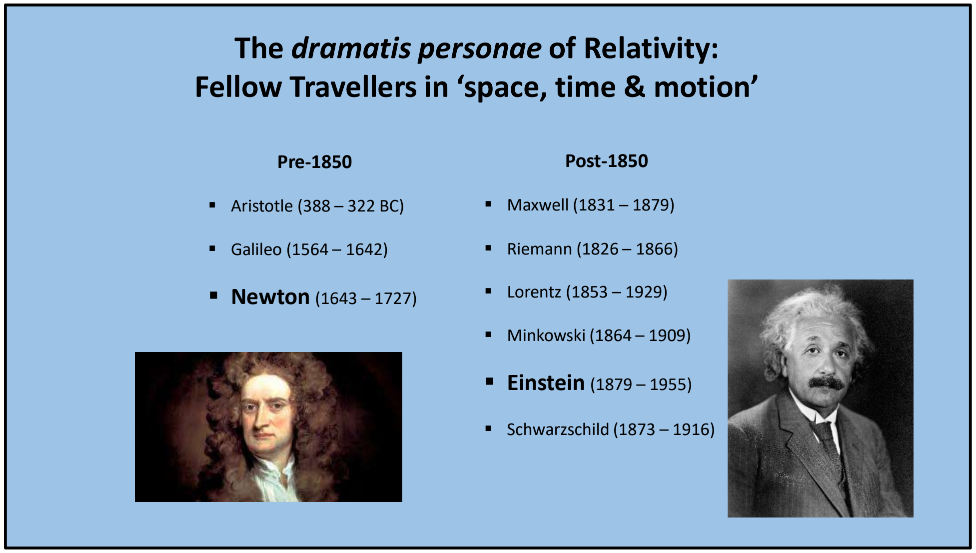
The Constant Speed of Light in a vacuum:

$$c = 3.0 \times 10^8 \text{ m/s}$$

or precisely: 299 792 458 m/s

[Speed of sound (in air at 20ºC) = 343 m/s]

- This is the constant speed of light in a vacuum. First determined accurately in the 19th century and found to be a constant value by the American's Michelson & Morley in 1887.
- It is believed presently that nothing can travel faster.
- If you are on a bike travelling at 25 m/s and direct a beam of light ahead, then the beam will not travel at (c + 25) m/s. This is the case whatever the speed of the cycle. If you increase the bike's speed to 0.5c then the combined speed of bike & beam will remain at c.
- Therefore very important number in SR and GR (& the Universe)
- Not to be confused with number 42 in 'Hitch hikers guide to the Galaxy' by Douglas Adams
- 186 000 miles/s



Pre-1850:

Aristotle: Early Greek natural philosopher, experimented with motion and gravity (not wholly successfully).
Galileo: working in Florence & Pisa during the Italian renaissance. First to use telescope (to study the moons of Jupiter, 8 then, about 80 now).
Also considered gravity and motion. Attempted to measure speed of light. Museum in Florence with much of his experimental equipment.
Newton: Another genius, of 16th-17th century England. Studied gravity and motion (& many other subjects in physics)

Post-1850:

Maxwell: 19th Century Scottish scientist who worked on electromagnetic radiation & the spectrum of waves (including light, atomic particles etc all travelling at c)
Laid the foundations for SR without knowing it. Nobel winner
Riemann: student of Gauss. The geometry of Euclid was unsuitable to describe the space of Einstein' relativity. Resorted

to Riemann geometry for ‘Spacetime’
Lorentz: Dutch scientist who introduced Lorentz transformations for frames of reference, and the Lorentz factor, gamma. Very important in relativity
Nobel winner
Minkowski: German who was Einstien’s maths lecturer at Zurich Polytechnic. Independently developed geometric method to describe 4d space &
published in 1908. Initially method was not thought of highly by Einstein at first, but proved essential for general relativity
Einstein: an uber-genius, a giant who stood on the shoulders of many giants. After graduating from Zurich Polytechnic in 1899, did not find academic post, and
working at Bern, Switzerland patent office. He thought very deeply about the work of his contemporaries and formulated his special theory incorporating
their latest ideas into a comprehensive theory in 1905. Nobel winner in 1921.
Schwarzschild: very important in solving Einstein’s equations of General Relativity for use with Black Holes in 1916. He died tragically in WWI a little later.

Einstein's Special Relativity : Key Points

- Speed of Light ($c = 3.0 \times 10^8$ m/s)
- 4-Dimensional Spacetime {t, x, y, z}
- Minkowski (Spacetime) Diagram
- Lorentz Transformation ($1 \leq \gamma < \infty$)
- Time Dilation
- Length Contraction
- Mass/Energy Equivalence ($E = mc^2$)
- Leading to General Relativity (1915)

Speed of light – already mentioned – is assumed in SR, & that it is fastest that objects can travel. One of the postulates of SR.
4d Spacetime {t,x,y,z}
Minkowski Spacetime diagram – geometric solutions of SR
Lorentz Transformation factor, gamma. Factor used to calculate time dilation and length contraction, as follows:
Time dilation or expansion: clocks run slow in **frames of reference** which are travelling relativisticly.
Length contraction: lengths are reduced in direction of travel at relativistic speeds.
$E = mc^2$: probably most famous equation. Mass-energy equivalence. 4th of the 1905 papers. Application of SR. Leads to nuclear power.
Add a few remarks on General Relativity – a theory of gravitation.

Dealing with Space and Time in Relativity

Galileo & Newton

Space & time are separate

Space: {x, y, z} 3 dimensions

Time: t absolute, independent

Distance: $d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$

where $\Delta x^2 = (x_1 - x_2)^2$ etc

Lorentz, Einstein & Minkowski

Integrated **'Spacetime'**

Space: {t, x, y, z} 4 dimensions

Time: relative, not constant

Interval: $\Delta s = \sqrt{-\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2}$

Differences in Relativity pre- and post-Einstein:
Space changes from 3d to 4d (including a time coordinate)
Time changes from being absolute to being relative. Eg. Time slows down in moving frames of reference.
Distance is originally given by 3d Pythagoras, but becomes a 4d 'interval' between two 'events'

Where is the use of Einstien’s Relativity Theory essential?

(i) **Particle Physics**: for example, at the Cern Large Hadron Collider (LHC).

(ii) **Global Positioning Systems**: precise timings and locations are essential.

(iii) **Astronomy**: for example, analysis of stars, in particular Black Holes.

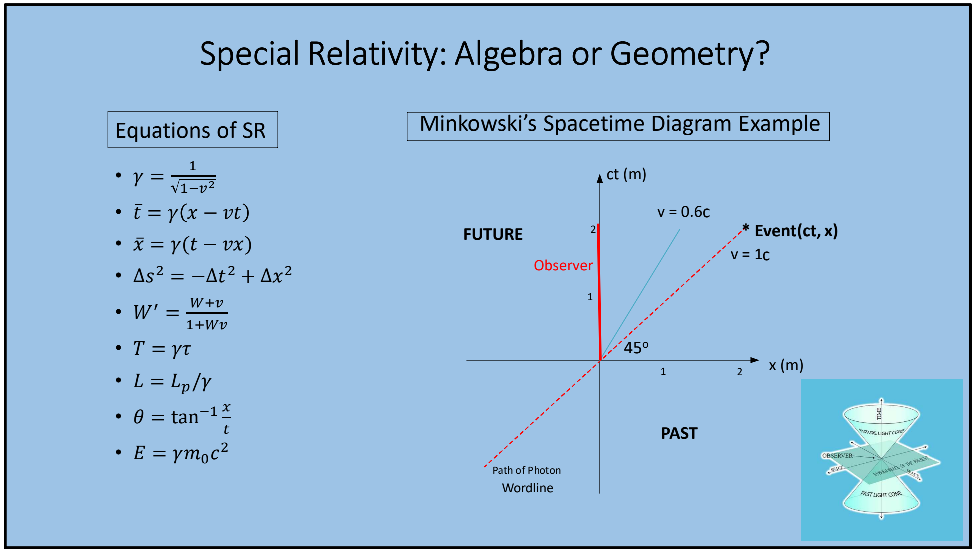
(iv) **Nuclear Power**: both civil & defence applications (fission & fusion)

Particle physics research at Cern: collision of protons at very high speeds in order to study the resulting particles eg. Higgs boson

GPS systems rely on precise timings and positions. This requires that adjustments be computed (using relativity) by the satellites used and by the receiver to minimize errors.

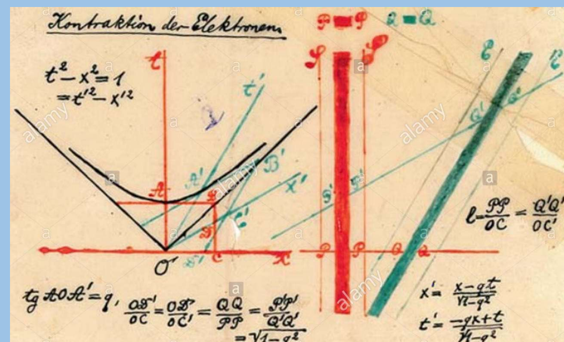
Astronomy: used for star evolution studies & the analysis of black holes

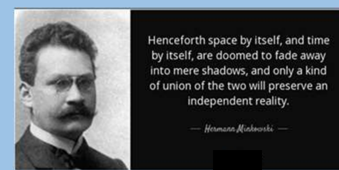
Nuclear Power: splitting of atoms



- There are 2 ways of understanding SR: through algebraic equations, or through the geometry of Minkowski spacetime diagrams.
- Initially when developing SR Einstein used the equation approach (& was not impressed by the geometric approach of Minkowski)
- Earlier Einstein was a maths student of Minkowski at Zurich Polytechnic
- He did however change his mind & used spacetime diagrams while working on GR.
- It is generally accepted that it is easier for most to understand SR using the geometric approach (together with a few equations).
- Looking at the spacetime diagram on the right: it is limited to 2 dimensions x and time (ct) in metres
- Red dotted line from L to R is path of photon at speed of light (v=c) at 45^O to 2 axes; path is upwards (travelling forward in time).

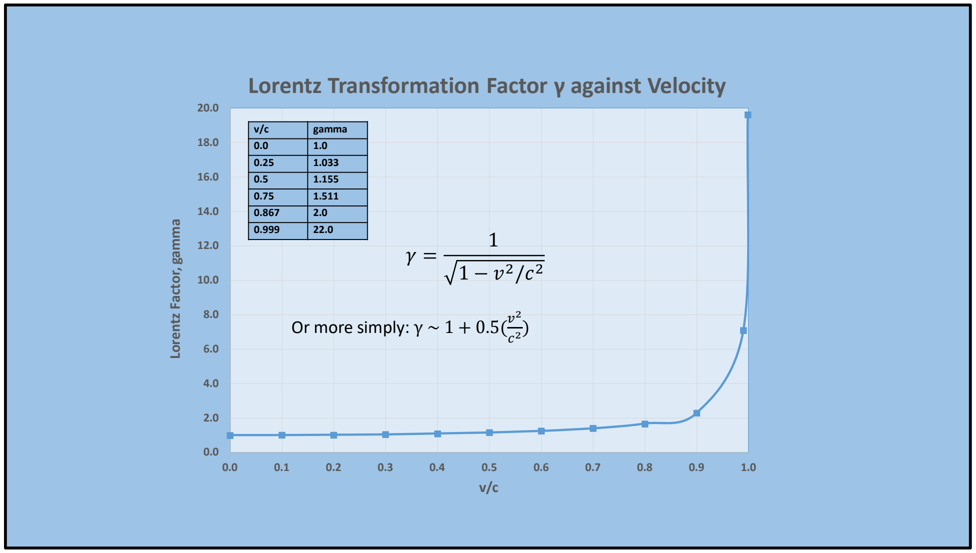
Minkowski's Spacetime (Raumzeit) Diagram





A slide (& quote) from Minkowski's presentation to a conference in Berlin 1908

Historical interest from 1908 conference in Berlin
Minkowski's 'Spacetime' diagram presentation slide (pre-PowerPoint)
Kontraktion der Electronem: Contraction of the Electron



Lorentz factor (gamma) against velocity/speed of light
Gamma used widely in SR to evaluate the passage of time, time dilation, & length contraction
At v/c = 0, gamma =1, at v/c = 1, gamma = infinity etc
Equation for gamma in centre, simple for calculations
At v/c = 0.5, gamma = 1.155 (small adjustments to mass, length and time)

0.867	2.0
0.999	22.0

Kink in graph is a bug in the MS Office package

Momentum (a measure of the motion of a body)
& use of Lorentz factor, gamma

Classical Physics: Momentum,

$$p = mv$$

m = mass, v = velocity

Relativistic Physics: Momentum,

$$p = \gamma m_0 v$$

γ = Lorentz factor
m_o = rest mass

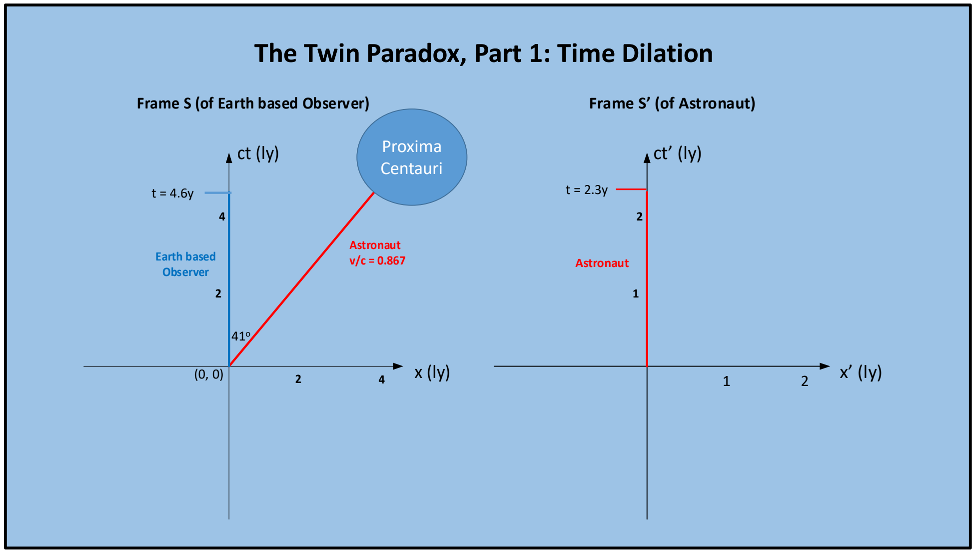
In SR mass **m** becomes $\boldsymbol{\gamma m_0}$, i.e. mass increases with speed of the body.

Example use of gamma in connection with momentum (Newton's 2nd Law)
Momentum is a measure of the motion of an object eg momentum = mass x velocity
Calculate gamma from v/c & use to find relativistic value (gamma > 1)
Mass increases with velocity
Gamma also used to adjust time and length

Combination of Velocities

Galilean Relativity	Special Relativity
	
$v_A = 8$ m/s ($\ll c$) $v_J = 24$ m/s ($\ll c$) Total velocity $= v_A + v_J$ $= 32$ m/s	$v_{CAR} = 31.3$ m/s ($\ll c$) $v_{BEAM} = 3.0 \times 10^8$ m/s ($= c$) $\text{Total Velocity} = \frac{(v_{car/c} + v_{beam}/c)c}{1 + v_{car}v_{beam}/c^2} = c$

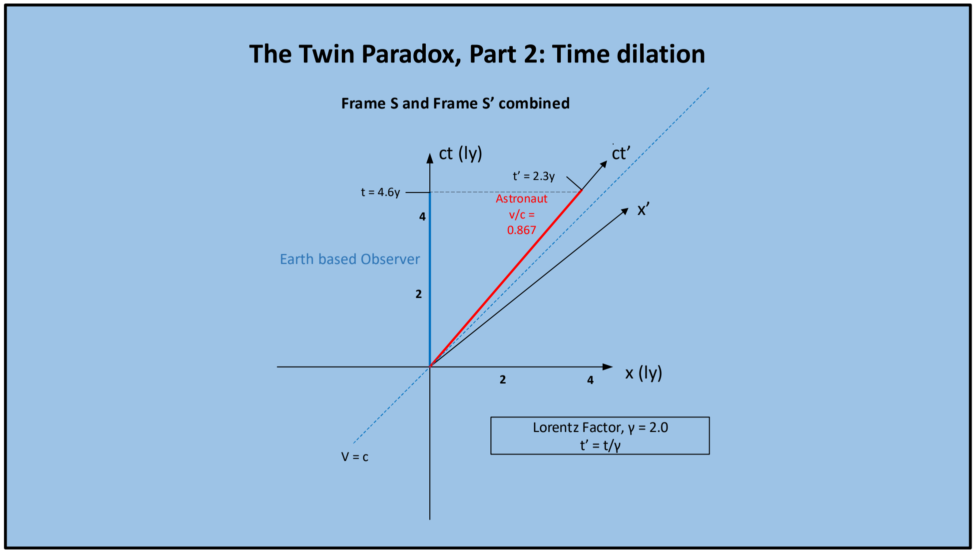
In pre-Einstein Relativity on the left (athlete throwing javelin) the total relative velocity is 8 + 24 = 32 m/s
After 1905 with special relativity on right hand side.
This shows a car travelling at 31.3 m/s with headlight beam travelling at speed of light c, total relative speed is NOT (31.3 + c), but just c.
An object cannot travel faster than the speed of light.



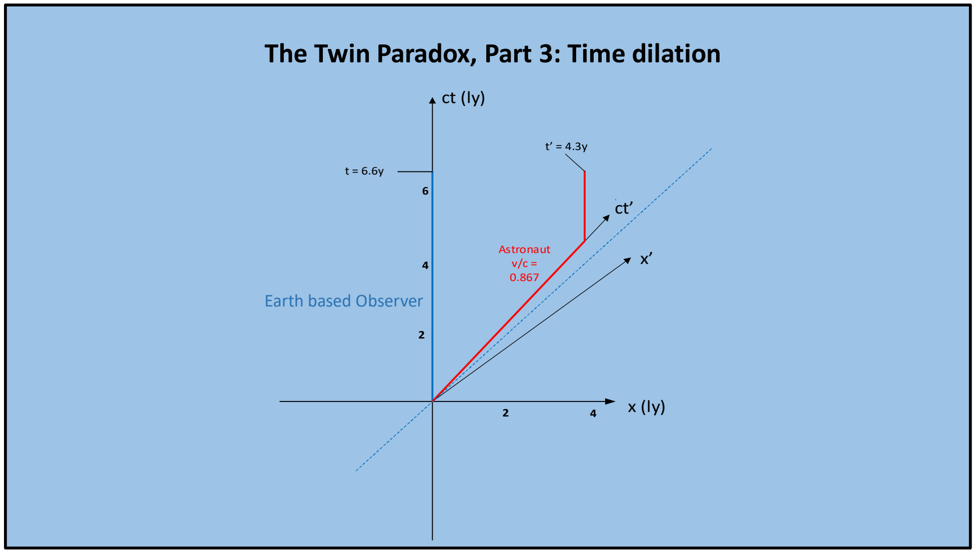
- The twin paradox is a well known illustration of **time dilation** & the implications of SR. It involves twins, one remaining on Earth (as an observer) & one (for example) travelling to our nearest star Proxima centauri, about 4.0 ly distant. This situation will be analysed using a series of Minkowski spacetime diagrams.
- Two frames of reference (where a reference frame is basically coordinate system with a set of clocks) one stationary on Earth & the other moving with the astronaut.
- The first diagram on the left is in the reference frame of the Earth, which shows the Earth based observer as the blue line on the ct axis, and the astronaut as the red line travelling at 0.867 times the speed of light towards the star. Gamma = 2.
- On the right is the spacetime diagram in the reference frame of the astronaut: the red vertical line on the ct' axis. In this reference frame the astronaut is stationary.
- Each frame has its own clock & these are synchronized at the common origin of each frame.
- These two diagrams may be combined so that the red line of the astronaut on the left becomes the ct' axis on the right, as in next slide.
- Notice that the scales of the 2 diagrams differ by factor of 2.
- Paradox: a seemingly absurd or contradictory statement or proposition which when investigated may prove

to be well founded or true

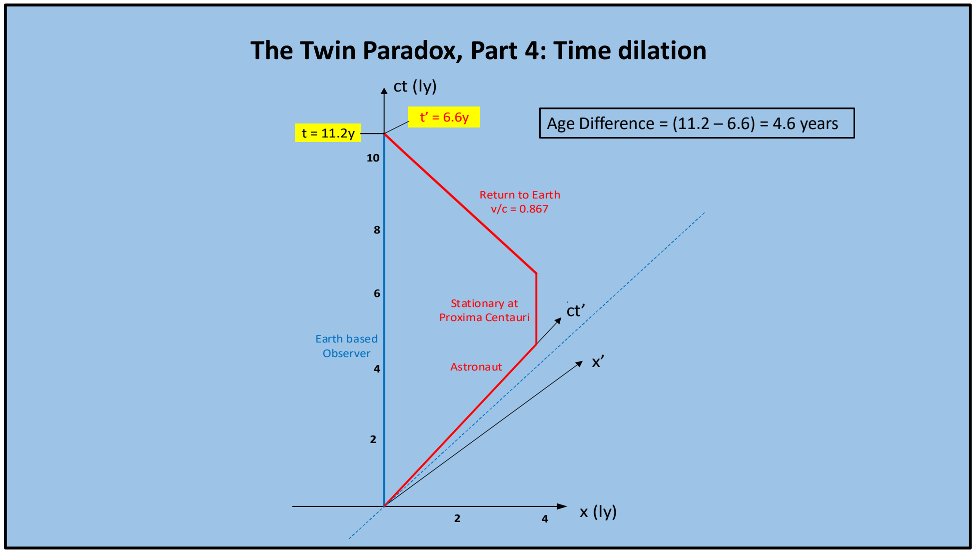
Light year is distance light travels in one year (c x 60 x 60 x 24 x 365m)



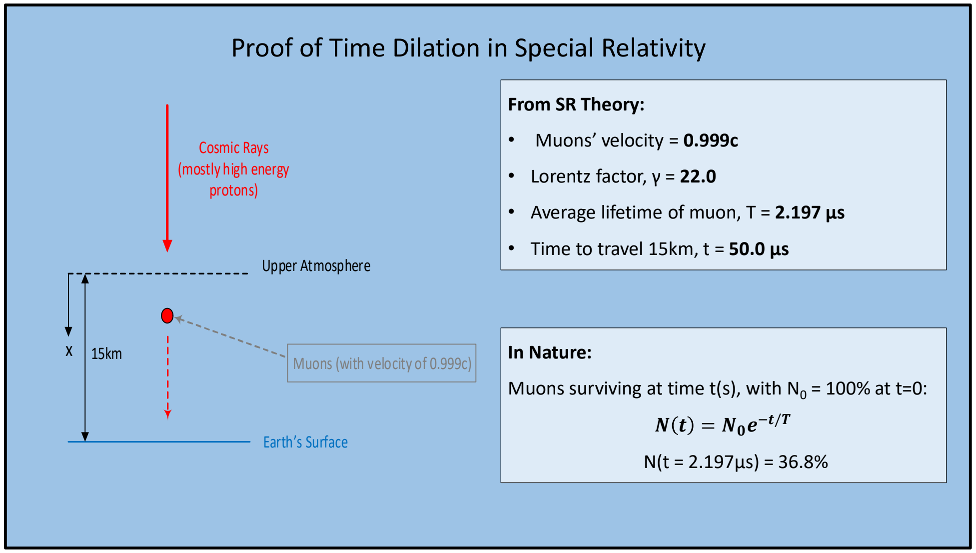
- This spacetime diagram shows the combined reference frames of previous slide: the ct & x axes remain in original positions, but the ct' & x' axes have been repositioned symmetrically about the dotted line of a photon c=1. These axes are uncalibrated at present.
- The clock on Earth will show an elapsed time of 4.6y. This is the time taken to travel a distance of 4 ly at velocity of 0.867c.
- However, in comparison, the clock on board the rocket will run slow, in line with the **time dilation** principle of SR. According to Lorentz transformation with a velocity of 0.867c, the Lorentz factor $\gamma = 2.0$ (chosen for convenience). If elapsed time on Earth (t) is 4.6 y, then elapsed time the astronaut experiences is half of this at 2.3 y.
- This is hard to accept at first, that time dilation is a true phenomenon, that time is not constant for the two observers. However it has been verified within the limits of the travel speeds possible today and for particles travelling at or near the speed of light. Of this more later.



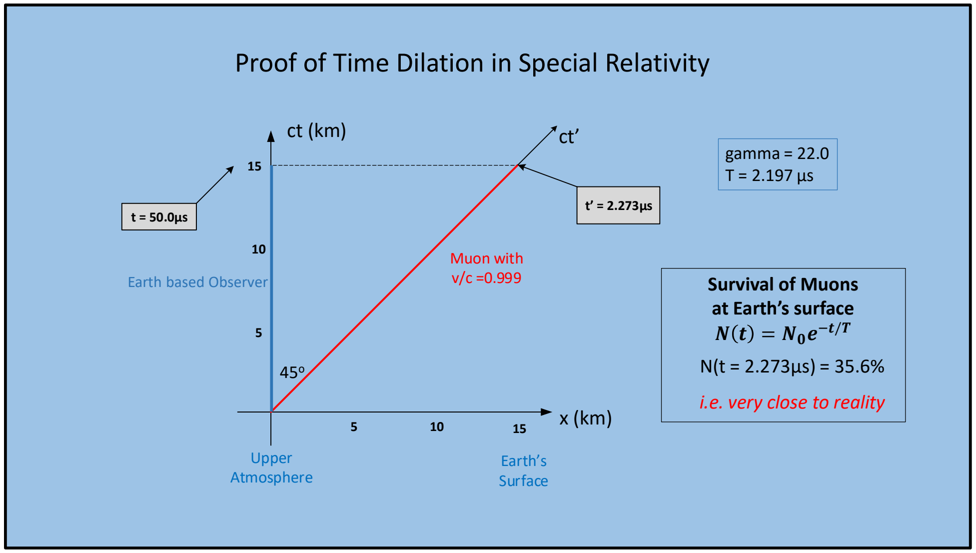
- The astronaut now stops for two years (possibly to carry out some research). The 'worldlines' of the 2 observers being extended (red & blue lines) upwards by 2 years, with no time dilation, because the position x of the astronaut remains constant.
- To date the elapsed time on Earth is 6.6 y, and the astronaut's clock will read 4.3 y
- The astronaut now returns to Earth (x = x' = 0).



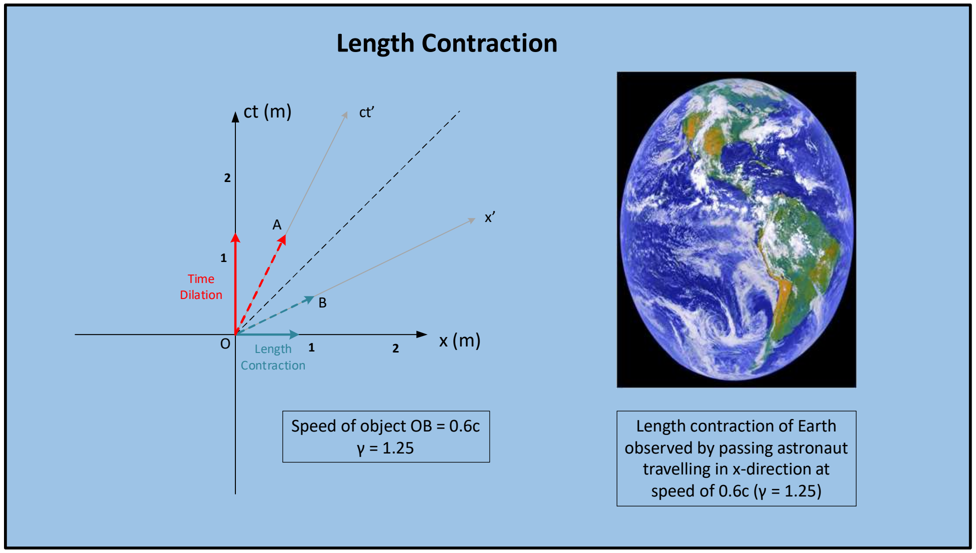
- On returning to Earth at similar velocity of 0.867c the previous arguments may be invoked to calculate the total elapsed times.
- The total time on Earth is 11.2 y, whereas the time on the rocket's clock will be 6.6 y.
- Therefore, if the age of the twins was 20 y at t = 0, then their ages will be 31.2 y and 26.6 y at the end of the voyage.
- This illustration may seem to be some sort of science-fiction, or even 'complete & utter codswallop'.
- At this time this example cannot be reproduced since modern space vehicles cannot travel at such high speeds. Max = 0.05c.
- However, all observations and experiments performed since 1905 have been able to verify time dilation in SR. A convincing example will follow.



An actual proof of SR and time dilation, involving the number of muons (like large electrons) reaching the Earth's surface.
Muons are formed when cosmic rays from outer space collide with the upper atmosphere of Earth (height of 15km)
Muons are like heavy electrons, travelling at 0.999c, with an average lifetime of T = 2.197 x 10^{-6} s
Lorentz factor of gamma ~22.0
Distance x = 0 at upper atmosphere
Time for muons to travel 15km is 50µs (see next slide)
In second box, using an exponential survival mathematical model, predicts 36.8% of muons will survive without splitting, leaving an electron
and 2 neutrinos remain.
Carry forward 50µs travel time & gamma to next slide.



Minkowski Spacetime diagram for Muon problem.
Distance x on horizontal axis ranges from 0.0km at upper atmosphere to 15.0km at Earth's surface.
Time ct on vertical axis (0-15km)
Axis scales are chosen so that red line (of muon travelling at 0.999c is approx. at 45deg to each axis.
Gamma = 22.0 from Lorentz factor formula.
Passage of time in observers frame is not the same as in muon's frame.
t=50µs calculated from ct of 15km (divide by 3 x 10^8 m/s)
For time in muon's frame divide by gamma, t' = t/gamma = 2.273µs (approx. average lifetime of a muon).
Predicted survival rate 35.6% at Earth's surface, as found by experiment.

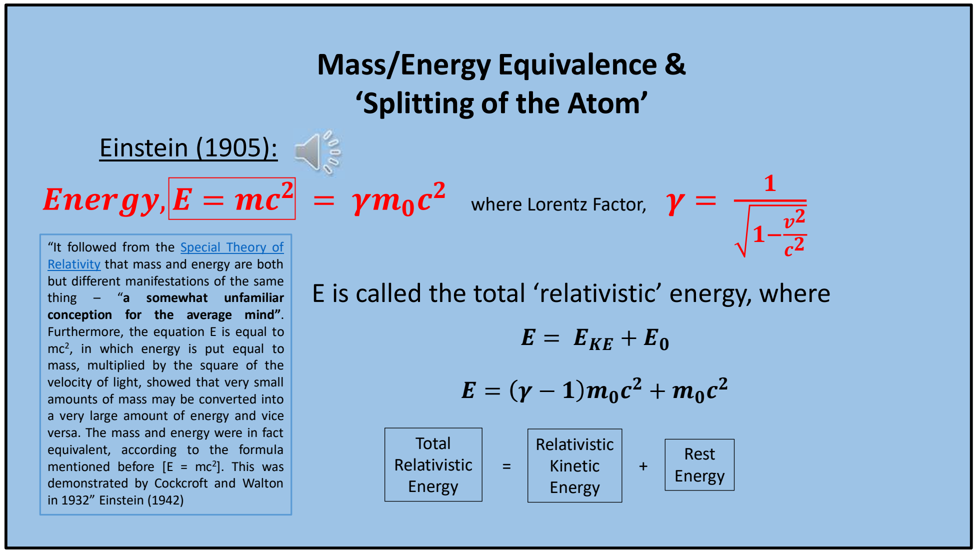


Length contraction is the phenomenon that a moving object's length is measured to be shorter than its proper length, which is the length as measured in the object's own rest length. It is also known as **Lorentz contraction**

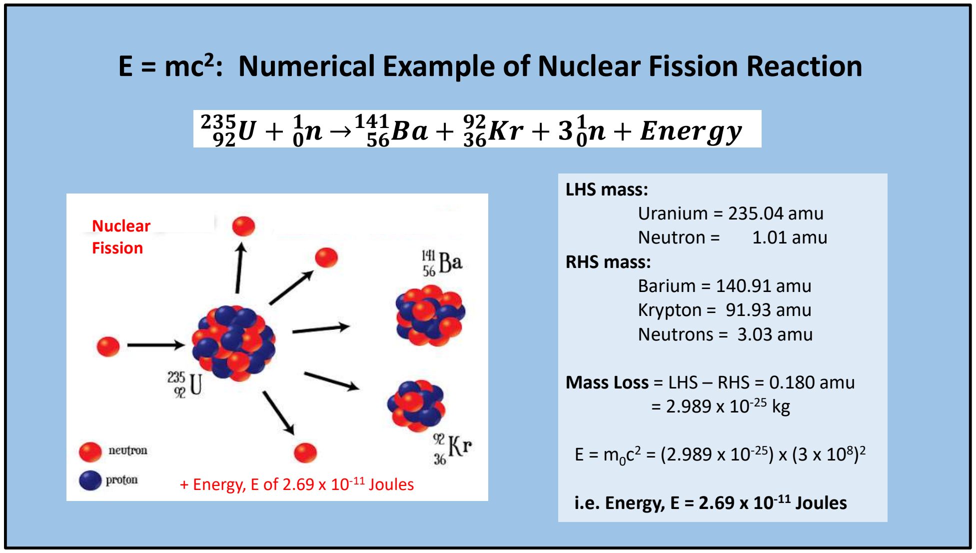
Spacetime diagram on the left showing both time dilation & length contraction

Speed of object 0.6c and gamma = 1.25

Right hand side shows what an astronaut at travelling at 0.6c in x direction would observe the Earth



Splitting of the atom, & mass/energy equivalence: Einstein's 2nd paper on application of Special Relativity in 1905
Very well known equation $E = mc^2$
Lorentz factor gamma appears in full equation along with m_0 the rest mass
Not until 1932 that an atom of lithium was split into 2 He atoms (ie. 2 alpha particles) at Cavendish Lab, Cambridge



Splitting the atom example
Chemical equation: 235 nucleus split by slow neutron
AMU – atomic mass unit 1/12 of mass of carbon12 nucleus
Slow & fast neutrons. Use of moderator eg Boron
Uranium 235 isotope, mass number, refinement, 238 predominates initially in mined uranium
Raw uranium is refined in centrifuges to increase the proportion of 235 atoms
Joule Nm

Einstein’s Field Equations of General Relativity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} - \Lambda g_{\mu\nu}$$

where

$G_{\mu\nu} =$ the **Einstein** tensor,

$R_{\mu\nu} =$ the **Ricci** tensor, a contraction of the **Riemann** tensor, $R^{\gamma}{}_{\mu\nu\gamma}$ given by:

$$R^{\delta}{}_{\mu\nu\gamma} = \frac{\partial \Gamma^{\delta}{}_{\mu\gamma}}{\partial x^{\nu}} - \frac{\partial \Gamma^{\delta}{}_{\mu\nu}}{\partial x^{\gamma}} + \Gamma^{\lambda}{}_{\mu\gamma} \Gamma^{\delta}{}_{\lambda\nu} - \Gamma^{\lambda}{}_{\mu\nu} \Gamma^{\delta}{}_{\lambda\gamma}$$

where the **Christoffel** coefficients are,

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\frac{\partial g_{\sigma\nu}}{\partial x^{\mu}} + \frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right)$$

The **Ricci** scalar R is given by contracting the **Ricci** tensor:

$$R = g^{\mu\nu} R_{\mu\nu}$$

and the metric tensor in spherical coordinates for **Schwarzschild** spacetime is,

$$g_{\mu\nu} = \begin{bmatrix} \frac{2GM}{c^2 r} - 1 & 0 & 0 & 0 \\ 0 & 1/\left(1 - \frac{2GM}{c^2 r}\right) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{bmatrix}$$

where $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$.

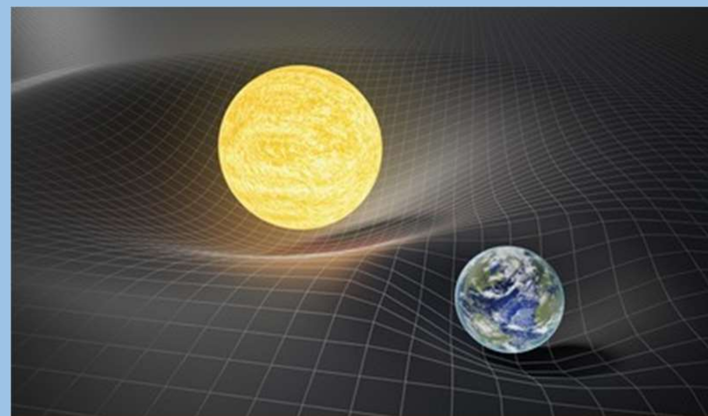
Finally on the right-hand side of the field equation $T_{\mu\nu}$ is the stress-energy-momentum tensor, equal to [0] for the **Schwarzschild** solution of a black hole, and the constant κ is the **Einstein** constant given by:

$$\kappa = \frac{8\pi G}{c^4}$$

where G is the gravitational constant, and c the speed of light in a vacuum. The second term on the right is a ‘dark energy’ term where Λ is the cosmological constant.

A few words about General Relativity (GR) 1915. One or two points worth pointing out.
Very much more complex than SR, as can seen from the equations in the slide
Einstein’s Field equations of GR (outlined in yellow) with G representing Einstein’s Tensor, a 4x4 matrix
Einstein was not a lone genius

2D representation of Spacetime in General Relativity



2d Spacetime represented by the grid shown, like the surface of a trampoline
Mass of Sun & Earth will depress the surface (normally flat) into curved spacetime.
Curved spacetime is considered to be gravity in GR
General Relativity is a theory of gravity to replace Newton’s ideas.
Gravity deflects light.
The bending of light around the sun was studied in the Sun’s complete eclipse of 1919
4d & 3d spacetime is very difficult to visualize
Gravity now defined as ‘Spacetime curvature’

Summary Remarks

- According to SR an object travelling at relativistic speeds is subject to significant changes to its mass, its length and the passage of time.
- Einstein's two Theories of Relativity, together with Quantum Mechanics (to which Einstein contributed) form the 3 most important pillars of modern physics.
- The ideas in the Special Theory are 'reasonably' easy to understand (either with mathematical or geometric methods), especially in comparison with the General Theory.
- In GR higher mathematical methods (eg. tensors) are required, although approximate methods are available (as exact solutions are rare).
- Relativity is essential in areas like, for example Nuclear Power, Particle Physics, Global Positioning Systems & Astronomical Analysis.

The final slide sums up some of the facts about Relativity we have been talking about today.